7/26/2007 SECOND HOURLY PRACTICE II Maths 21a, O. Knill, Summer 2007

Name:

- Start by printing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

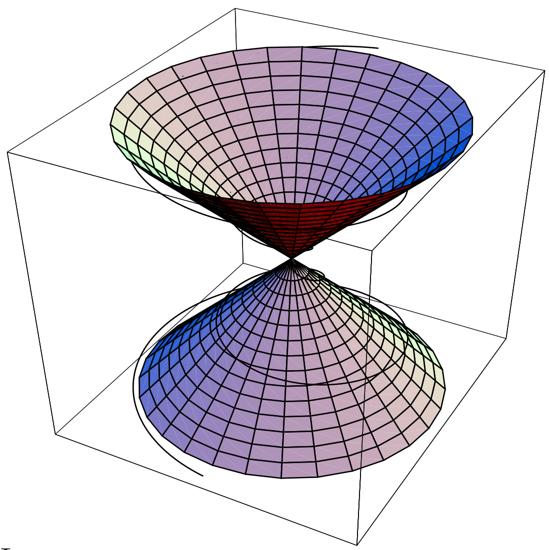
Problem 1) True/False questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.

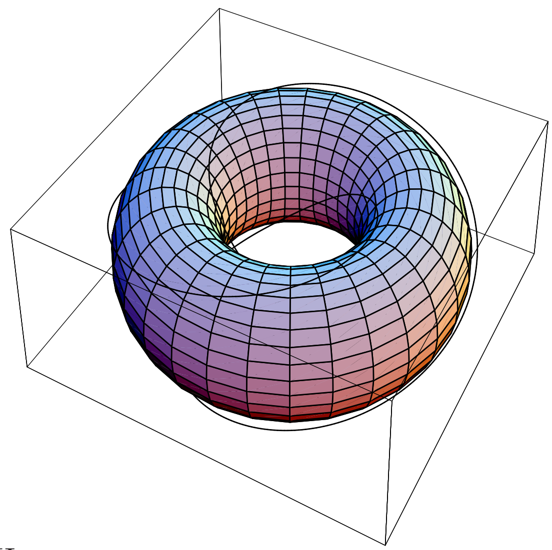
1)	T	F	The function $f(t,x,y) = (y+t)\cos(x-t)$ satisfies the partial differential equation $f_{tt} = f_{xx} + f_{yy}$ which is called the two dimensional wave equation.
2)	T	F	The velocity of the curve $\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle$ at time $t = 0$ is 1.
3)	T	F	There exists a function $f(x,y)$ of two variables which every point (n,m) with integer n and integer m is a critical point.
4)	T	F	If $f_x(x,y) = f_y(x,y) = f_{xx}(x,y) = f_{yy}(x,y) = f_{xy}(x,y) = 0$ for all (x,y) then $f(x,y) = 0$ for all (x,y) .
5)	T	F	$(0,0)$ is a local minimum of the function $f(x,y) = x^6 + y^6$.
6)	T	F	If $f(x,y)$ has a local max at the point $(0,0)$ with discriminant $D > 0$, then $g(x,y) = f(x,y) - x^9 + y^9$ has a local max at $(0,0)$ too.
7)	T	F	The value of the function $f(x,y) = \sqrt{1+3x+5y}$ at $(-0.002, 0.01)$ can by linear approximation be estimated as $1 - (3/2) \cdot 0.002 + (5/2) \cdot 0.01$.
8)	T	F	The curve $\vec{r}(t) = (x(t), y(t), z(t)) = (\sin(t)t^5, \sin(t)t^5, \sin(t)t^5)$ is a line in space.
9)	T	F	The chain rule can be written in the form $\frac{d}{dt} f(\vec{r}(t)) = D_{\vec{r}\,'(t)} f(\vec{r}(t))$
10)	T	F	The curvature of the curve $\vec{r}(t) = (\cos(5t), \sin(5t)$ for $t = 0$ is 5 times the curvature of the curve $r(t) = (\cos(t), \sin(t))$ for $t = 0$.
11)	T	F	The gradient of f at a point (x_0, y_0, z_0) is tangent to the level surface of f which contains (x_0, y_0, z_0).
12)	T	F	If the Lagrange multiplier λ is positive, then the critical point under constraint is a local minimum.
13)	T	F	If the directional derivative $D_{\vec{v}} f(1,1) = 0$ for all vectors $\vec{v}$, then $(1,1)$ is a critical point of $f(x,y)$.
14)	T	F	The arc length of the curve $r(t) = \langle t, \cos(t) \rangle$ from $t = 0$ to $t = 2\pi$ is the integral $\int_0^{2\pi} \sqrt{t^2 + \cos^2(t)}\, dt$.
15)	T	F	For any curve $\vec{r}(t)$, the vectors $\vec{r}\,''(t)$ and $\vec{r}\,'(t)$ are always perpendicular to each other.
16)	T	F	Every critical point (x,y) of a function $f(x,y)$ for which the discriminant D is not zero is either a local maximum or a a local minimum.
17)	T	F	The function $f(x,y) = e^y x^2 \sin(y^2)$ satisfies the partial differential equation $f_{xxyyyxyy} = 0$.
18)	T	F	If $(0,0)$ is a critical point of $f(x,y)$ and the discriminant D is zero but $f_{xx}(0,0) < 0$ then $(0,0)$ can not be a local minimum.
19)	T	F	In the second derivative test, one can replace the condition $D > 0, f_{xx} > 0$ with $D > 0, f_{yy} > 0$ to check whether a point is a local minimum.
20)	T	F	The arc-length of $\vec{r}(t) = (0,0,1) + t(0,3,4)$ with $t \in [0,2]$ is 10.

Problem 2) (10 points)

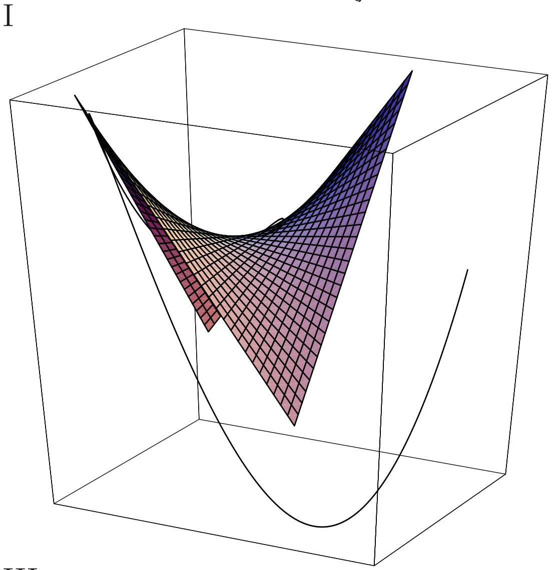
Match the parameterizations with the curves. In this problem, the curves are located on parameterized surfaces with two parmeters, but of course, the curves themselves have only one parameter. No justifications are needed.



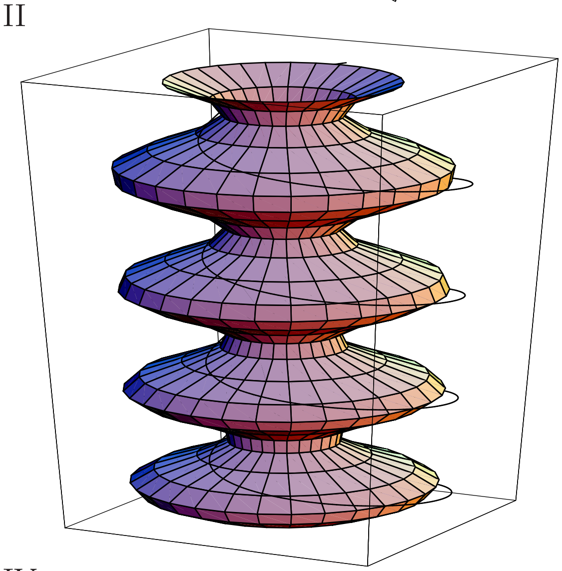
I



II



III



IV

Enter I,II,III,IV here	Parameterization
	$\vec{r}(t) = (t\sin(t), t\cos(t), t)$
	$\vec{r}(t) = (2+\cos(2t)\cos(3t), (2+\cos(2t))\sin(3t), \sin(2t))$
	$\vec{r}(t) = (t\cos(t), t\sin(t), t^2\cos(t)\sin(t))$
	$\vec{r}(t) = ((2+\sin(t))\sin(t), (2+\sin(t))\cos(t), t)$

Problem 3) (10 points)

Tell from each of the 10 following objects, whether they are a vector or a scalar. No justifications are needed. Each correct answer is 1 point. All objects are defined for objects in space $\mathbf{R}^3$.

object	vector	scalar
curvature		
arc length		
velocity		
speed		
acceleration		
unit tangent vector		
jerk		
discriminant		
directional derivative		
gradient		

Problem 4) (10 points)

Find all the critical points of

$$f(x,y) = x^3 + y^3 - 3x - 12y$$

and indicate whether they are local maxima, local minima or saddle points.

Problem 5) (10 points)

When Ramanujan, an amazing mathematician who was born in India was sick in the hospital and the English mathematician Hardy visited him, Ramanujan asked "whats up?" Hardy answered. "Nothing special. Even the number of the taxi cab was boring: 1729". Ramanujan answered: "No, that is a remarkable number. It is the smallest number, which can be written in two different ways as a sum of two perfect cubes. Indeed $1729 = 1^3 + 12^3 = 9^3 + 10^3$.

a) (5 points) Find the linearization $L(x, y, z)$ of the function $f(x, y, z) = x^3 + y^3 - z^3$ at the point $(9, 10, 12)$.

b) (5 points) Use the technique of linear approximation to estimate $9.001^3 + 10.02^3 - 12.001^3$.

Problem 6) (10 points)

a) (5 points) Find the equation $ax + by + cz = d$ for the tangent plane to the level surface

$$f(x, y, z) = x^3 + y^3 - z^3 = 1$$

at the point $(1, 1, 1)$. Note that this is the same Ramanujan function as in the previous problem.

b) (5 points) If we intersect the level surface $f(x, y, z) = 1$ with the plane $z = 2$, we obtain the equation for an implicit curve $x^3 + y^3 = 9$. It is a level curve for the function $g(x, y) = x^3 + y^3$. Find the tangent line to this curve at the point $(1, 2)$.

Problem 7) (10 points)

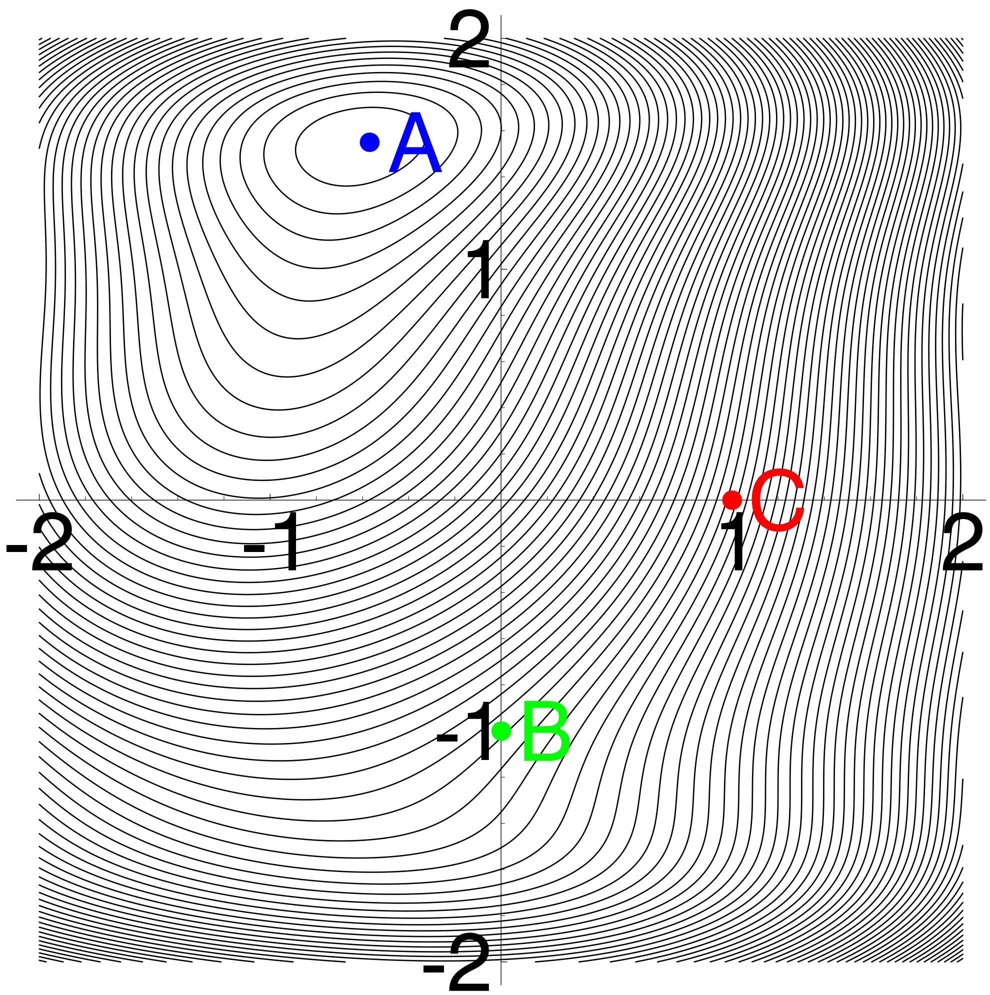
A function $f(x, y)$ of two variables describes the height of a mountain. You don't know the function but you see its level curves. The mountain has its peak at the point A on the picture.

a) (3 points) At which point does f take its maximum under the constraint $x = 0$.

b) (3 points) At which point does f take its maximum under the constraint $y = 0$.

c) (4 points) At which of the points B or C is the length of the gradient vector larger?

Note: As always, you have to give explanations to get full credit. The points in a) and b) are not necessarily marked. Give it up to an accuracy of 1/2. For example, an answer in a) or b) could look like $(0.5, 1.5)$.



Problem 8) (10 points)

Olivers great-grand-dad Emil Frech Hoch (1874-1947) founded the car company "Frech-Hoch" in Switzerland. The company produces cars and trucks. The company still exists today and produces specialized vehicles.

Assume the revenue of the company is $f(x, y) = x^2 + 2y^2$, where x is the number of cars and y is the number of trucks produced per year. The production is constrained by the amount of steel available. Trucks need twice as much steel leading to $g(x, y) = x + 2y = 1$. Use the Lagrange multiplier method to find the optimal production rate.

Problem 9) (10 points)

During a bus ride over the country side, a kid throws an unfinished apple out of the window at time $t = 0$. The bus drives at that time with a speed of 40 (meters per second) in the x direction. The apple is thrown from the hand with an initial velocity of $(0, 1, 1)$ meters per second from a height of 4 meters. We chose a coordinate system such that the initial position of the apple is $\vec{r}(0) = (0, 0, 4)$. You can assume the acceleration of the earth is $(0, 0, -10)$ meters per seconds^2.

a) (4 points) Find the parameterization of the path $\vec{r}(t) = (x(t), y(t), z(t))$ of the apple after the drop.

b) (3 points) At which time does it hit the earth, if one assumes the apple falls without being slowed down by the air resistance?

c) (3 points) Where does it hit the earth?