7/12/2007 FIRST MIDTERM **Maths 21a, O. Knill, Summer 2007**

Name:

- Start by writing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have exactly 90 minutes to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) (20 points) No justifications are needed.

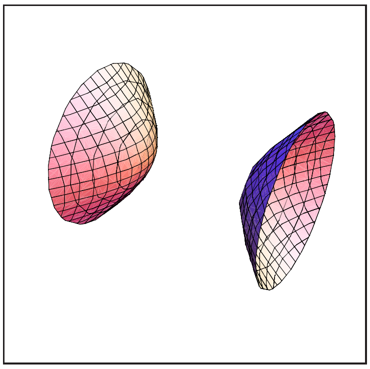
1)	T	F	The vector $\vec{v} = \langle 1, -1, 0 \rangle$ is perpendicular to the line $1 - x = 1 - y = 1 - z$.
2)	T	F	The length of the vector $\vec{v} = \langle 1, 0, -1 \rangle$ is 2.
3)	T	F	$\langle 1, 2, 3 \rangle \times \langle 2, 4, 6 \rangle = \langle 0, 0, 0 \rangle$.
4)	T	F	The distance between a point P and a line through the origin O can not be larger than $\|OP\|$.
5)	T	F	The vectors $\vec{u} = \langle 1, -1, 0 \rangle$ and $\vec{PQ}$ with $P = (1, 1, 1)$ and $Q = (2, 2, 2)$ are parallel.
6)	T	F	The surface $y^2 - z^2 = 1$ is an elliptical paraboloid.
7)	T	F	If two vectors $\vec{v}, \vec{w}$ are orthogonal, then their cross product is the zero vector.
8)	T	F	The surface $x^2 + y^2 - z^2 = 2x$ is called a one-sheeted hyperboloid.
9)	T	F	The set of points which have distance 1 from the x-axis is a cylinder.
10)	T	F	If in spherical coordinates a point is given by $(\rho, \theta, \phi) = (1, \pi, \pi/2)$, then its Euclidean coordinates are $(x, y, z) = (-1, 0, 0)$.
11)	T	F	The volume of a parallelepiped spanned by $(1, 1, 1), (1, 0, 0)$ and $(0, 1, 2)$ is equal to 1.
12)	T	F	The two planes $x + y - z = 1$ and $-2x - 2z + 2z = 5$ intersect in a line.
13)	T	F	In cylindrical coordinates, the equation $r^2 = z$ defines a paraboloid.
14)	T	F	The vector projection of the vector $(1, 1, 1)$ onto the vector $\langle 0, 2, 0 \rangle$ is $\langle 0, 1, 0 \rangle$.
15)	T	F	The point given in cylindrical coordinates as $(r, \theta, z) = (1, \pi/2, 1)$ is in Cartesian coordinates the point $(x, y, z) = (0, 1, 1)$.
16)	T	F	If $f = g(x, y)$ is a graph then $\vec{r}(u, v) = (u, v, g(u, v))$ is a parameterization of the surface.
17)	T	F	There is a function $f(x, y, z)$ such that $f(x, y, z) = 1$ is a hyperboloid and $f(x, y, z) = -1$ is a paraboloid.
18)	T	F	The projection of $\vec{v} = \langle 1, 1, 1 \rangle$ onto the vector $\langle 1, 0, 0 \rangle$ is $\langle \sqrt{3}, 0, 0 \rangle$.
19)	T	F	The points satisfying $(x - 1)^2 - (y - 1)^2 + (z + 1)^2 = 1$ form a hyperboloid
20)	T	F	The equations $x = y = z$ describe a line which contains the vector $\langle 1, 1, 1 \rangle$.

Total

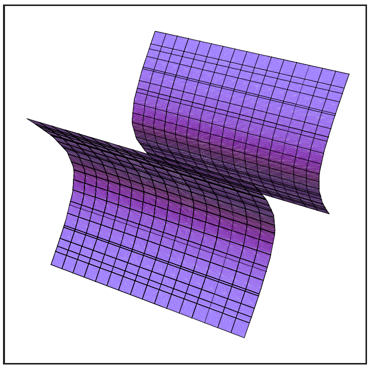
Problem 2) (10 points)

Match the equations $g(x, y, z) = d$ with the surfaces.

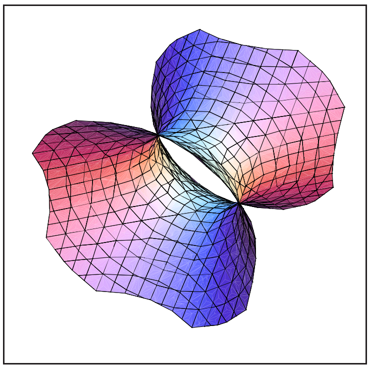
I



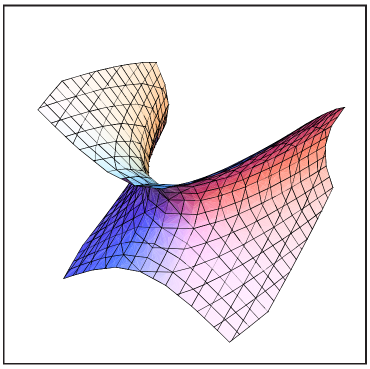
II



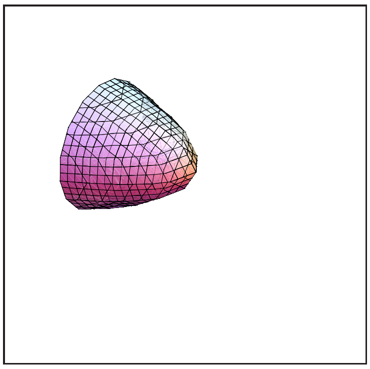
III



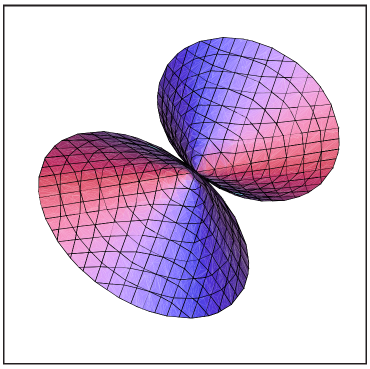
IV



V



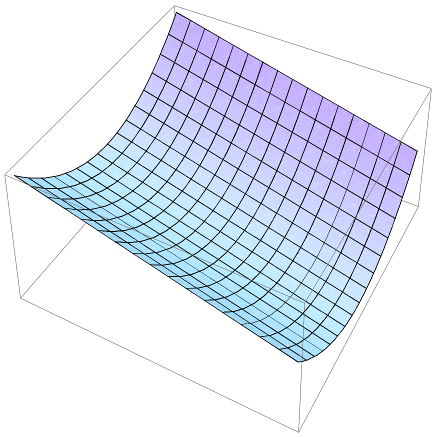
VI



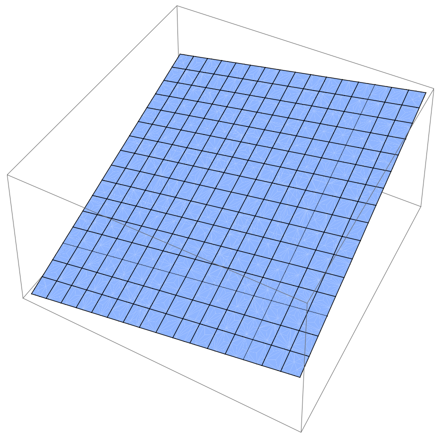
Enter I,II,III,IV,V,VI here	Equation
	$x^2 + y^2 - z^2 = 0$
	$x^2 - y^2 - z^2 = 1$
	$x^2 - y^2 + z^2 = 1$
	$x^2 - y - z^2 = 1$
	$x^2 - z^2 = 1$
	$x + y^2 + z^2 = 0$

Problem 3) (10 points)

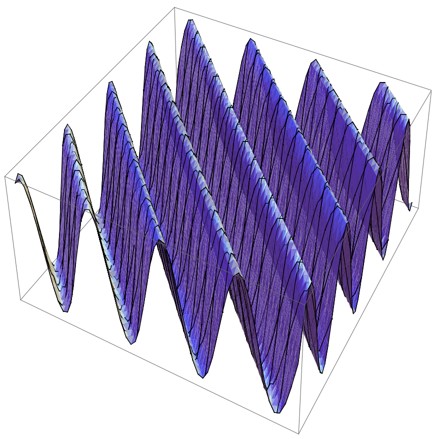
Match the functions with their graphs. No justifications are needed.



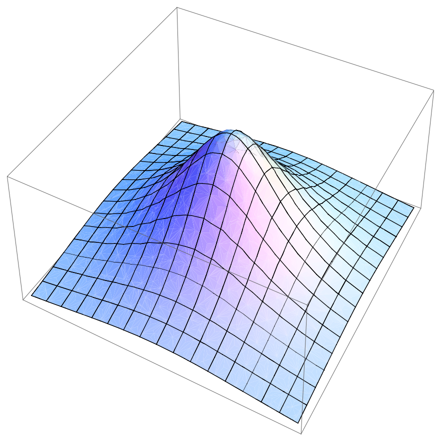
I



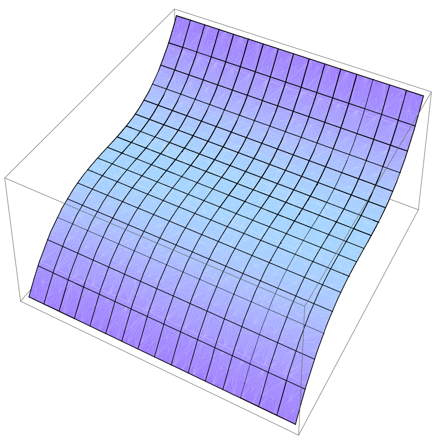
II



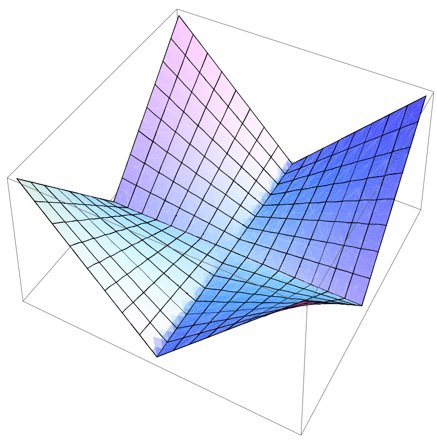
III



IV



V



VI

Enter I,II,III,IV,V or VI here	Equation
	$z = 3x + 5y + 1$
	$z = x/(1 + x^2 + y^2)$
	$z = y^2 - x$
	$z = \|x\| \cdot \|y\|$
	$z = \sin(6(x + y))$
	$z = y^3$

Problem 4) (10 points)

Au usual, $\text{proj}_{\vec{v}}(\vec{w})$ is the vector projection of $\vec{w}$ onto the vector $\vec{v}$ and $\text{comp}_{\vec{v}}(\vec{w})$ is the scalar projection. Compute:

a) $(4,5,1) \cdot (1,-1,1)$

b) $(-1,1,3) \times (1,1,1)$

c) $(2,1,3) \cdot ((3,4,5) \times (1,1,3))$.

d) $\vec{\text{proj}}_{\langle 1,0,0\rangle}\langle 7,3,2\rangle$

e) $|\langle 0,3,4\rangle| + \text{comp}_{\langle 1,0,0\rangle}\langle 3,4,5\rangle$

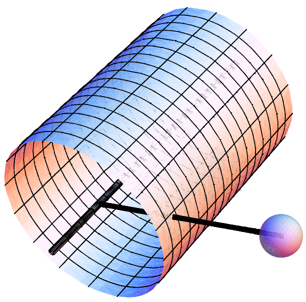
Problem 5) (10 points)

We want to find the distance between the point $P = (5,0,-5)$ and the cylinder which has an axes going through the points $A = (1,1,1)$ and $B = (0,2,1)$ and radius 1. To do so:

a) (4 points) Find first a parameterization $\vec{r}(t) = Q + t\vec{v}$ of the line.

b) (4 points) Find the distance between P and the line.

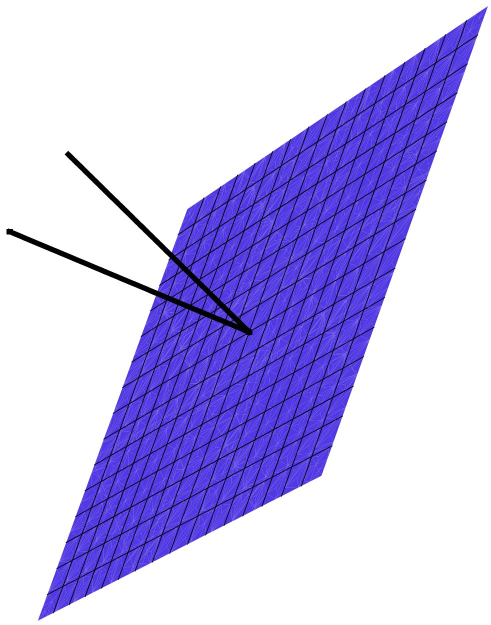
c) (2 points) Now find the distance between P and the cylinder.



Problem 6) (10 points)

The angle between a line and a plane is defined as $\pi/2 - \alpha$, if α is the angle between the normal vector to the plane and a vector in the line.

Find the angle between the plane $x + y - z/2 = 1$ and the line $\vec{r}(t) = \langle 1, 1, 1 \rangle + t\langle 1, 1, 3 \rangle$.



Problem 7) (10 points)

Find the implicit equation

$$ax + by + cz = d$$

of the plane which contains the line

$$\vec{r}(t) = \vec{r}_0 + t\vec{v} = \langle -1, 1, 1 \rangle + t\langle 3, 4, 5 \rangle$$

and the point $Q = (7, 7, 9)$.

Problem 8) (10 points)

If you cut the plane $x + 3y + 2z = 6$ by the coordinate planes, we get a triangle.

a) (4 points) What are the vertices A, B, C of this triangle?

b) (6 points) Find the area of this triangle.

Problem 9) (10 points)

Complete in the following table with missing parameterizations, implicit descriptions or graphs. Each line is a specific surface.

parametric $\vec{r}(u,v) =$	implicit	graph	name
		$z = x^2 + y^2$	
$\langle \sin(v)\cos(u), \sin(v)\sin(u), \cos(v) \rangle$			upper half sphere
	$x^2 - y^2 + z = 0$		
$\langle 1,1,1 \rangle + u\langle 1,0,1 \rangle + v\langle 0,1,1 \rangle$			