Name:

- Start by writing your name in the above box.

- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.

- Do not detach pages from this exam packet or unstaple the packet.

- Please write neatly. Answers which are illegible for the grader can not be given credit.

- No notes, books, calculators, computers, or other electronic aids can be allowed.

- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) (20 points) No justifications are needed.

|		
T	**F**	The vector $\vec{v} = (1, 2, -4)$ is perpendicular to the plane $4x + 2y + 2z = 100$.
T	F	With $\vec{i} = (1, 0, 0), \vec{j} = (0, 1, 0), \vec{k} = (0, 0, 1)$, the formula $(\vec{i} \times \vec{j}) \times (\vec{j} \times \vec{i}) = \vec{0}$ holds.
T	F	The equations $x/5 = y/7 = z/8$ describe a line which contains the origin $(0, 0, 0)$.
T	**F**	The vectors $\vec{u} = (3, -2, 1)$ and $\vec{OQ}$ with $O = (0, 0, 0)$ and $Q = (-6, 4, 2)$ are parallel.
T	F	If $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$ are perpendicular, then the vectors $\vec{u}$ and $\vec{v}$ have the same length.
T	F	The two vectors $(2, 3, 0)$ and $(6, -4, 5)$ are orthogonal.
T	**F**	For any two vectors $\vec{v}, \vec{w}$, one has $|\vec{v} + \vec{w}|^2 = |\vec{v}|^2 + |\vec{w}|^2$.
T	F	The surface $x^2 - y^2 + z^2 = 1$ is called a one-sheeted hyperboloid.
T	F	The set of points which have distance 2 from the x-axis is a cylinder.
T	F	If in spherical coordinates a point is given by $(\rho, \theta, \phi) = (2, \pi/2, \pi/2)$, then its Euclidean coordinates is $(x, y, z) = (0, 2, 0)$.
T	F	If the distance of a point to a line is zero, then the point is on the line.
T	F	A surface which is given as $r = 2 + \sin(z)$ in cylindrical coordinates stays the same when we rotate it around the z axis.
T	**F**	The length $|\vec{v} - \vec{w}|$ of the difference $\vec{v} - \vec{w}$ of two parallel vectors $\vec{v}, \vec{w}$ is always equal to the difference $|\vec{v}| - |\vec{w}|$ of the lengths of the vectors.
T	**F**	The volume of a parallelepiped spanned by $(1, 0, 0), (0, 1, 0)$ and $(1, 1, 1)$ is equal to $1/3$.
T	**F**	The equation $x^2 = y + z^2$ describes an elliptic paraboloid.
T	F	In spherical coordinates, the equation $\cos(\theta) = \sin(\theta)$ is the plane $x - y = 0$.
T	**F**	If $|\vec{v} \times \vec{w}| = 0$ then $\vec{v} = \vec{0}$ or $\vec{w} = \vec{0}$ or $\vec{v} = \vec{w}$.
T	F	The vector projection of the vector $(1, 1, 1)$ onto the vector $(0, 2, 0)$ is $(0, 1, 0)$.
T	**F**	The point given in spherical coordinates as $(\rho, \theta, \phi) = (\sqrt{8}, 3\pi/2, \pi/2)$ is in Cartesian coordinates the point $(x, y, z) = (2, -2, 0)$.
T	**F**	If $g(x, y, z) = 0$ is an implicit equation, then $\vec{r}(u, v) = (u, v, g(u, v, g(u, v)))$ is a parametrization of the surface.

	Total

Problem 2) (10 points)

Match the equations $g(x, y, z) = d$ with the surfaces.

I

II

III

IV

V

VI
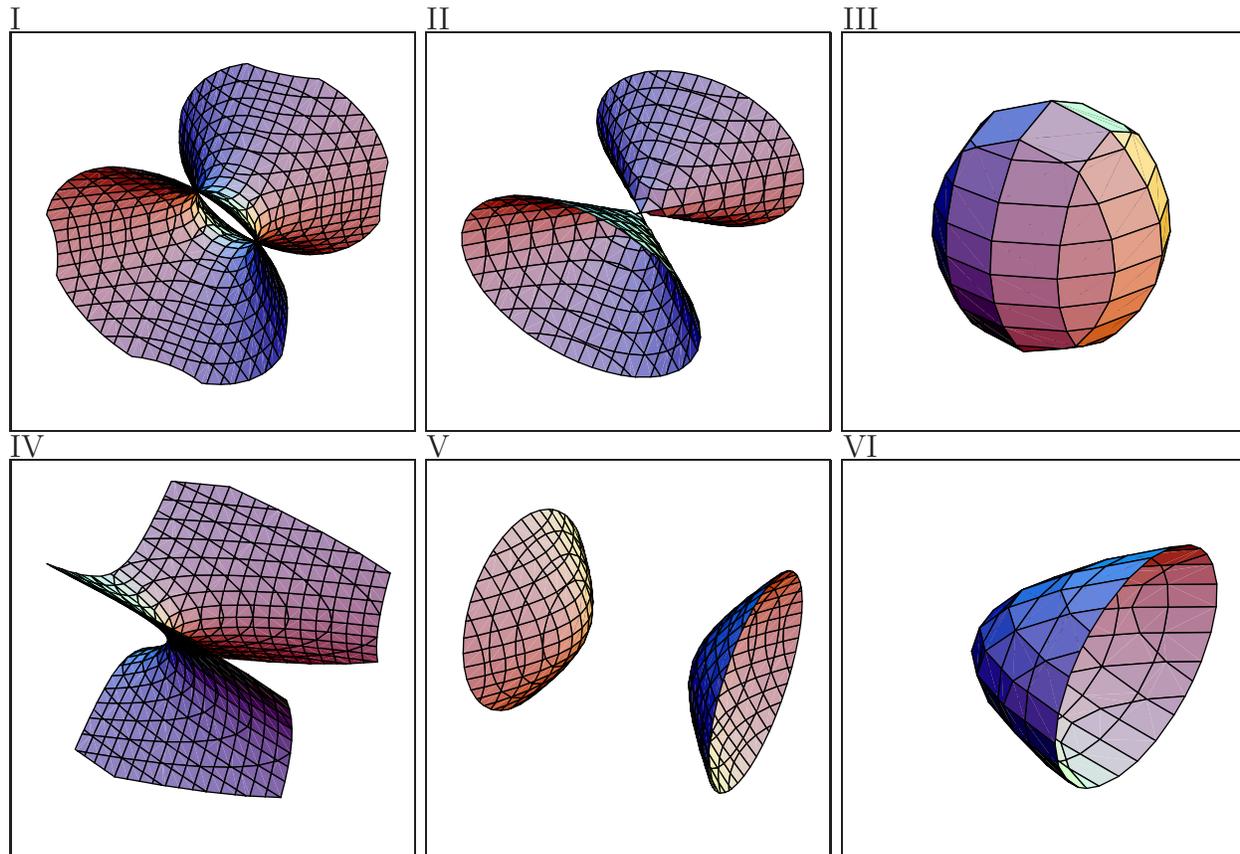
Enter I,II,III,IV,V,VI here	Equation
	$x^2 + 2z^2 - y^2 = 0$
	$y^2 + z^2 + 4x^2 = 1$
	$y^2 - z^2 - x^2 = -1$
	$x^2 - y^2 - z^2 = 1$
	$y^2 - z^2 - x = 1$
	$-y^2 - z^2 + x = 1$

Solution:

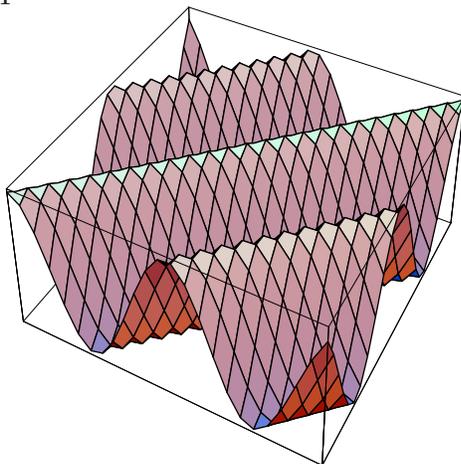| Enter I,II,III,IV,V,VI here	Equation
II	$x^2 + 2z^2 - y^2 = 0$
III	$y^2 + z^2 + 4x^2 = 1$
I	$y^2 - z^2 - x^2 = -1$
V	$x^2 - y^2 - z^2 = 1$
IV	$y^2 - z^2 - x = 1$
VI	$-y^2 - z^2 + x = 1$

Problem 3) (10 points)

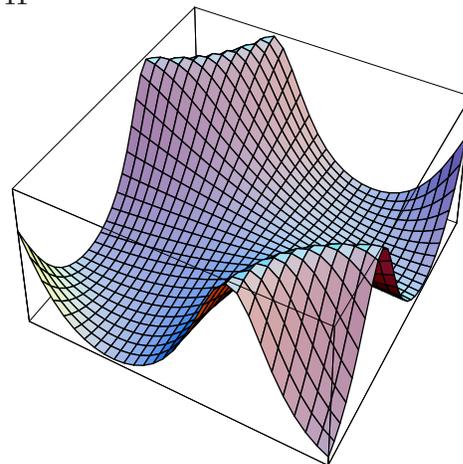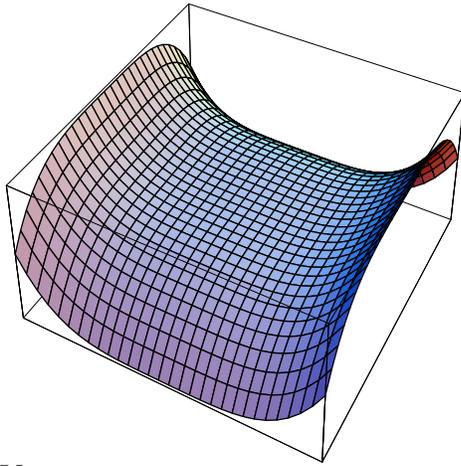Match the equation with their graphs. No justifications are needed.

I

II

III

IV

V
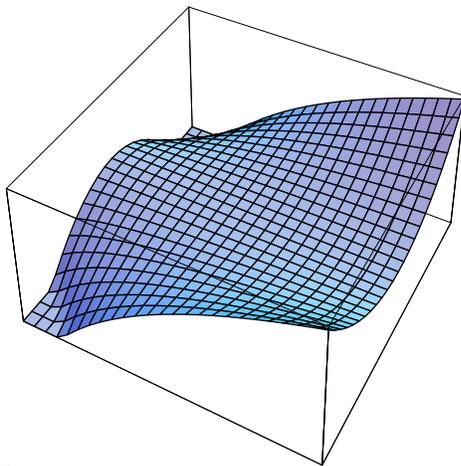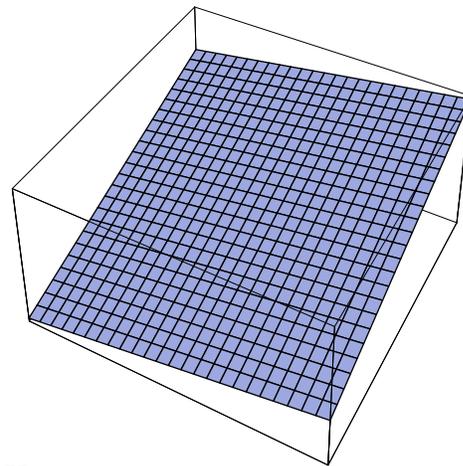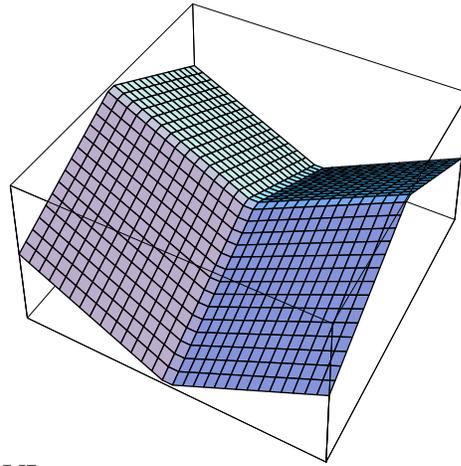
VI

Enter I,II,III,IV,V or VI here	Equation
	$z = |x| - |y|$
	$z = y^2 \log(x)$
	$z = \cos(5(x - y))$
	$z = x + 2y$
	$z = x/(2 + \sin(xy))$
	$z = x^4 - y^4$

Solution:

Enter I,II,III,IV,V or VI here	Equation
VI	$z = |x| - |y|$
I	$z = y^2 \log(x)$
III	$z = \cos(5(x - y))$
II	$x + 2y$
IV	$z = x/(2 + \sin(xy))$
V	$z = x^4 - y^4$

Problem 4) (10 points)

Reminder. $\text{proj}_{\vec{v}}(\vec{w})$ is the vector projection of $\vec{w}$ onto the vector $\vec{v}$. $\text{comp}_{\vec{v}}(\vec{w})$ denotes the scalar projection. Compute:

a) $(7, 5, 1) \cdot (1, -1, 1)$

b) $(-1, 1, 3) \times (1, 1, 1)$

c) $(2, 1, 3) \cdot ((3, 4, 5) \times (1, 1, 3))$.

d) $\vec{\text{proj}}_{(1,0,0)}(7, 3, 2)$

e) $\text{comp}_{(0,1,0)}(7, 3, 2)$

Solution:
a) 3.
b) $(-2, 4, -2)$.
c) $((3, 4, 5) \times (2, 1, 3)) = (7, -4, -1)$ and the result is 7.
d) This is a vector projection: $(7, 0, 0)$.
e) This is a scalar projection: 3.

Problem 5) (10 points)

Find the distance between the point $P = (1, 0, -1)$ and the line which contains the points $A = (1, 1, 1)$ and $B = (0, 2, 1)$.
To do so:

a) (4 points) Find first a parametrization $\vec{r}(t) = Q + t\vec{v}$ of the line.

b) (6 points) Now find the distance.

Solution:
a) $\vec{v} = \vec{AB} = ((0, 2, 1) - (1, 1, 1)) = (-1, 1, 0)$ is in the line. ' A parametrization is $\vec{r}(t) = (1, 0, -1) + t(-1, 1, 0)$. We can write it as $\vec{r}(t) = (1 - t, t, -1)$.
b) Use the distance formula: $|\vec{AP} \times \vec{v}|/|\vec{v}| = |(0, -1, -2) \times (-1, 1, 0)|/|(-1, 1, 0)| = \boxed{3/\sqrt{2}}$.

Problem 6) (10 points)

The angle between two planes is defined as the angle between the two normal vectors of the planes. Given the planes $x - z = 1$ and $y + z = 3$.

a) (2 point) Find the normal vector $\vec{v}$ to the first plane and the normal $\vec{w}$ to the second plane.

b) (2 points) Find the angle α between the two planes.

c) (2) points) Verify that $P = (4, 0, 3)$ and $Q = (1, 3, 0)$ are on the intersection of the two planes.

d) (2 points) Find a parametrization of the line of intersection of the two planes.

e) (2 points) Find the symmetric equation of the line of intersection of the two planes.

Solution:
a) $\vec{v} = (1, 0, -1)$ and $\vec{w} = (0, 1, 1)$. b) $\cos(\alpha) = \vec{v} \cdot \vec{w}/(|\vec{v}||\vec{w}|) = -1/2$. So, $\alpha = 2\pi/3$.
c) Plug into $(x, y, z) = (4, 0, 3)$ and $(x, y, z) = (1, 3, 0)$ into the equations. d) $P + t(P-Q) = (1, 2 + t, 2 - 3t)$.
e) $(x - 1)/1 = (y - 2)/1 = -(z - 2)/3$.

Problem 7) (10 points)

Find the implicit equation

$$ax + by + cz = d$$

of the plane which contains the line

$$\vec{r}(t) = P + t\vec{v} = (-1, 1, 1) + t(3, 4, 5)$$

and the point $Q = (7, 7, 9)$.

Solution:
The vector $\vec{v} = (3, 4, 5)$ as well as the vector $\vec{PQ} = (8, 6, 8)$ are in the plane. Their cross product $\vec{v} \times \vec{PQ} = (2, 16, -14)$ is perpendicular to the plane. The plane has therefore the equation $2x + 16y - 14z = d$. The constant $d = 0$ can be obtained by pluggin in a point. Fro example the point $(-1, 1, 1)$. The final answer is $2x + 16y - 14z = 0$.

Problem 8) (10 points)

a) (6 points) Find the surface area of the parallelepiped which contains the points $A = (0, 0, 0)$, $B = (1, 1, 0)$, $C = (0, 1, 1)$, $D = (1, 0, 1)$.

b) (4 points) Find the volume of the solid.
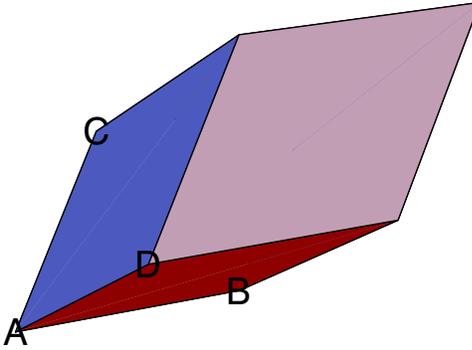
Solution:

a) The surface of the parallelepiped has 6 faces. Each of the faces can be computed with a cross product. If $\vec{u} = \vec{AB} = (1,1,0)$, $\vec{v} = \vec{AC} = (0,1,1)$ and $\vec{w} = \vec{AD} = (1,0,1)$, then the area is $2|\vec{u} \times \vec{v}| + 2\vec{u} \times \vec{w} + 2\vec{v} \times \vec{w} = 2|(1,-1,1)||2|(1,-1,-1)| + 2|(1,1,-1)| = 2\sqrt{3} + 2\sqrt{3} + 2\sqrt{3} = \boxed{6\sqrt{3}}$.

b) The volume is the triple scalar product of $\vec{u}$, $\vec{v}$ and $\vec{w}$. It is $|(1,1,0) \cdot (1,1,-1)| = \boxed{2}$.

Problem 9) (10 points)

a) (4 points) We define a scalar valued function which has as argument two vectors:

$$f(\vec{v}, \vec{w}) = \frac{|\vec{v} \times \vec{w}|}{\vec{v} \cdot \vec{w}}$$

What is $f((1,0,0), (1,1,0))$?

b) (6 points) The function $f(\vec{v}, \vec{w})$ turns out to be a function of the angle α between $\vec{v}$ and $\vec{w}$ only. What is this function?

Solution:

a) The cross product is $(0,0,1)$. The dot product is 1. The result is $1/1 = \boxed{1}$.

b) Write $|\vec{v} \times \vec{w}| = |\vec{v}||\vec{w}| \sin(\alpha)$ and $\vec{v} \cdot \vec{w} = |\vec{v}||\vec{w}| \cos(\alpha)$. The quotient is $\sin(\alpha)/\cos(\alpha) = \boxed{\tan(\alpha)}$.