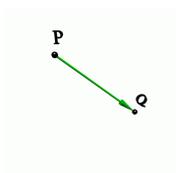


**DISTANCES**

DISTANCE POINT-POINT (3D). If  $P$  and  $Q$  are two points, then

$$d(P, Q) = |\vec{PQ}|$$

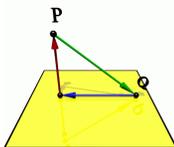
is the distance between  $P$  and  $Q$ .



DISTANCE POINT-PLANE (3D). If  $P$  is a point in space and  $\Sigma : \vec{n} \cdot \vec{x} = d$  is a plane containing a point  $Q$ , then

$$d(P, \Sigma) = |(\vec{PQ}) \cdot \vec{n}| / |\vec{n}|$$

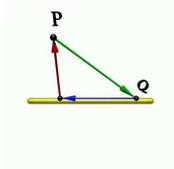
is the distance between  $P$  and the plane.



DISTANCE POINT-LINE (3D). If  $P$  is a point in space and  $L$  is the line  $\vec{r}(t) = Q + t\vec{u}$ , then

$$d(P, L) = |(\vec{PQ}) \times \vec{u}| / |\vec{u}|$$

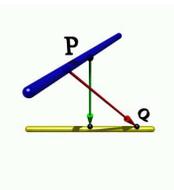
is the distance between  $P$  and the line  $L$ .



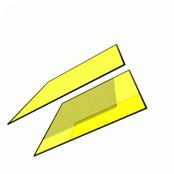
DISTANCE LINE-LINE (3D).  $L$  is the line  $\vec{r}(t) = Q + t\vec{u}$  and  $M$  is the line  $\vec{s}(t) = P + t\vec{v}$ , then

$$d(L, M) = |(\vec{PQ}) \cdot (\vec{u} \times \vec{v})| / |\vec{u} \times \vec{v}|$$

is the distance between the two lines  $L$  and  $M$ .



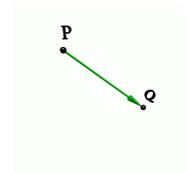
DISTANCE PLANE-PLANE (3D). If  $\vec{n} \cdot \vec{x} = d$  and  $\vec{n} \cdot \vec{x} = e$  are two parallel planes, then their distance is  $(e - d) / |\vec{n}|$ . Nonparallel planes have distance 0.



**EXAMPLES**

DISTANCE POINT-POINT (3D).  $P = (-5, 2, 4)$  and  $Q = (-2, 2, 0)$  are two points, then

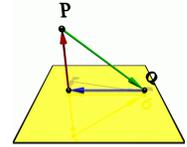
$$d(P, Q) = |\vec{PQ}| = \sqrt{(-5 + 2)^2 + (2 - 2)^2 + (0 - 4)^2} = 5$$



DISTANCE POINT-PLANE (3D).  $P = (7, 1, 4)$  is a point and  $\Sigma : 2x + 4y + 5z = 9$  is a plane which contains the point  $Q = (0, 1, 1)$ . Then

$$d(P, \Sigma) = |(-7, 0, -3) \cdot (2, 4, 5)| / |\sqrt{45}| = 29 / \sqrt{45}$$

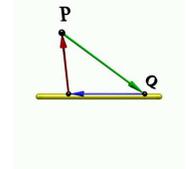
is the distance between  $P$  and  $\Sigma$ .



DISTANCE POINT-LINE (3D).  $P = (2, 3, 1)$  is a point in space and  $L$  is the line  $\vec{r}(t) = (1, 1, 2) + t(5, 0, 1)$ . Then

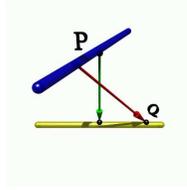
$$d(P, L) = |(-1, -2, 1) \times (5, 0, 1)| / \sqrt{26} = |(-2, 6, 10)| / \sqrt{26} = \sqrt{140} / \sqrt{26}$$

is the distance between  $P$  and  $L$ .



DISTANCE LINE-LINE (3D).  $L$  is the line  $\vec{r}(t) = (2, 1, 4) + t(-1, 1, 0)$  and  $M$  is the line  $\vec{s}(t) = (-1, 0, 2) + t(5, 1, 2)$ . The cross product of  $(-1, 1, 0)$  and  $(5, 1, 2)$  is  $(2, 2, -6)$ . The distance between these two lines is

$$d(L, M) = |(3, 1, 2) \cdot (2, 2, -6)| / \sqrt{44} = 4 / \sqrt{44}$$



DISTANCE PLANE-PLANE (3D).  $5x + 4y + 3z = 8$  and  $5x + 4y + 3z = 1$  are two parallel planes. Their distance is  $7 / \sqrt{50}$ .

