7/28/2005 SECOND HOURLY **Maths 21a, O. Knill, Summer 2005**

Name:

- Start by printing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

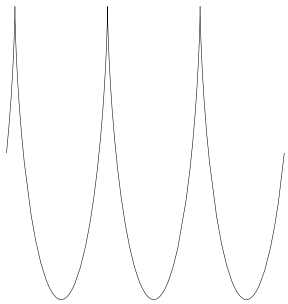
Problem 1)	(20 points)

T	F	The function $f(t,x,y) = \sin(x-t)y$ satisfies the partial differential equation $f_{tt} = (f_{xx} + f_{yy})$.
T	F	The acceleration of the curve $\vec{r}(t) = (\cos(t), \sin(t), t)$ at time $t = 0$ is 1.
T	F	There exists a function $f(x,y)$ of two variables which has no critical points at all.
T	F	If $f_x(x,y) = f_y(x,y) = 0$ for all (x,y) then $f(x,y) = 0$ for all (x,y) .
T	F	$(0,0)$ is a local maximum of the function $f(x,y) = x^2 - y^2 + x^4 + y^4$.
T	F	If $f(x,y)$ has a local maximum at the point $(0,0)$ with discriminant $D > 0$ then $g(x,y) = f(x,y) - x^4 + y^3$ has a local maximum at the point $(0,0)$ too.
T	F	The value of the function $f(x,y) = \sqrt{1+3x+5y}$ at $(-0.002, 0.01)$ can by linear approximation be estimated as $1 - (3/2) \cdot 0.002 + (5/2) \cdot 0.01$.
T	F	The curve $\vec{r}(t) = (x(t), y(t)) = (t^3, t^3)$ is a line in the plane
T	F	The directional derivative $D_{\vec{v}} f$ is a vector, which is normal to $\vec{v}$.
T	F	The gradient of f at a point (x_0, y_0, z_0) is tangent to the level surface of f which contains (x_0, y_0, z_0).
T	F	The sign of the Lagrange multiplier λ tells you whether the extrema under constraint is a local maximum or local minimum.
T	F	If $D_{\vec{v}} f(1,1) = 0$ for all vectors $\vec{v}$, then $(1,1)$ is a critical point of $f(x,y)$.
T	F	The vector $\nabla f(1,1,1)$ is perpendicular to the surface $f(x,y,z) = x^2 + y^2 + 2z^2 = 4$ at the point $(1,1,1)$.
T	F	The function $u(x,t) = x^3 + t^3$ satisfies the wave equation $u_{tt} = u_{xx}$.
T	F	For any curve $\vec{r}(t)$, the vectors $\vec{r}\,''(t)$ and $\vec{r}\,'(t)$ are always perpendicular to each other.
T	F	Every critical point (x,y) of a function $f(x,y)$ for which the discriminant D is not zero is either a local maximum or a a local minimum.
T	F	The function $f(x,y) = e^y x^2 \sin(y^2)$ satisfies the partial differential equation $f_{xxyyyxyy} = 0$.
T	F	If $(0,0)$ is a critical point of $f(x,y)$ and the discriminant D is zero but $f_{xx}(0,0) < 0$ then $(0,0)$ can not be a local minimum.
T	F	In the second derivative test, one can replace the condition $D > 0, f_{xx} > 0$ with $D > 0, f_{yy} > 0$ to check whether a point is a local minimum.
T	F	The length of the curve $\vec{r}(t) = (0,0,1) + t(0,3,4)$ with $t \in [0,2]$ is equal to 10.

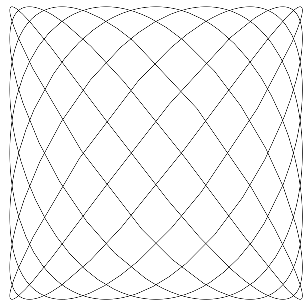
Space for work

Problem 2) (10 points)

Match the parameterizations with the curves. No justifications are needed.



I



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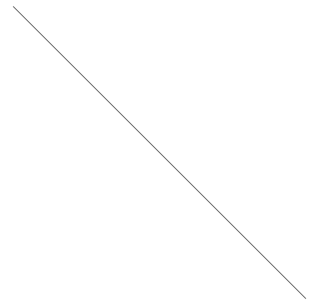

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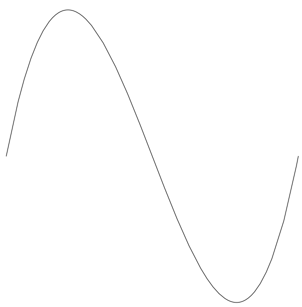

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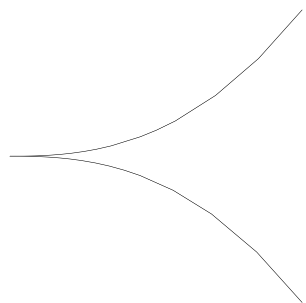

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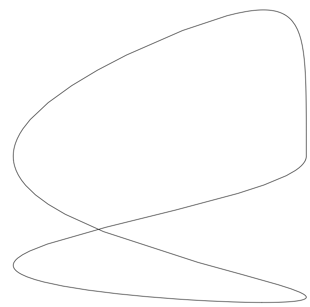

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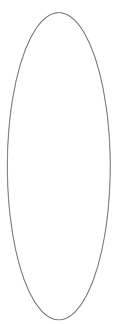

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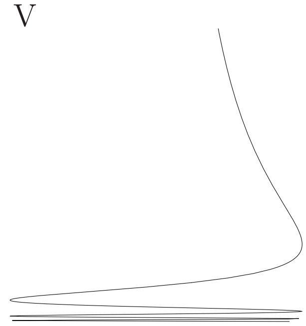
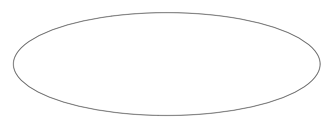


VIII



IX

Enter I,II,..., until IX here	Parametrization	Parameter interval $[a, b]$.
	$\vec{r}(t) = (\cos(7t), \sin(9t))$	$[0, 2\pi]$
	$\vec{r}(t) = (t, 1 - t)$	$[-1, 1]$
	$\vec{r}(t) = (t, t^3 - t)$	$[-1, 1]$
	$\vec{r}(t) = (t^2, t^5)$	$[-1, 1]$
	$\vec{r}(t) = (\cos(t), 3\sin[t])$	$[0, 2\pi]$
	$\vec{r}(t) = (\cos(\pi t^2), \sin(\pi t))$	$[0, 2]$
	$\vec{r}(t) = (\sin(1/t), t)$	$[0, \pi]$
	$\vec{r}(t) = (3\cos(t), \sin(t))$	$[0, 2\pi]$
	$\vec{r}(t) = (\cos(t) + t, \sin(t))$	$[0, 6\pi]$

Space for work

Problem 3)	(10 points)

Tell from each of the 10 following objects, whether they are a vector or a scalar. No justifications are needed. Each correct answer is 1 point.

object	vector	scalar
velocity		
speed		
acceleration		
arc length		
curvature		
discriminant		
directional derivative		
gradient		
partial derivative		

Space for work

Problem 4) (10 points)

Find all the critical points of $f(x, y) = \frac{x^5}{5} - \frac{x^2}{2} + \frac{y^3}{3} - y$ and indicate whether they are local maxima, local minima or saddle points.

Space for work

Problem 5) (10 points)

Use the technique of linear approximation to estimate $f(0.003, -0.0001, \pi/2 + 0.01)$ for

$$f(x, y, z) = \cos(xy + z) + x + 2z \ .$$

Space for work

Problem 6) (10 points)

Find the equation $ax + by + cz = d$ for the tangent plane to the level surface of

$$f(x, y, z) = \cos(xy + z) + x + 2z$$

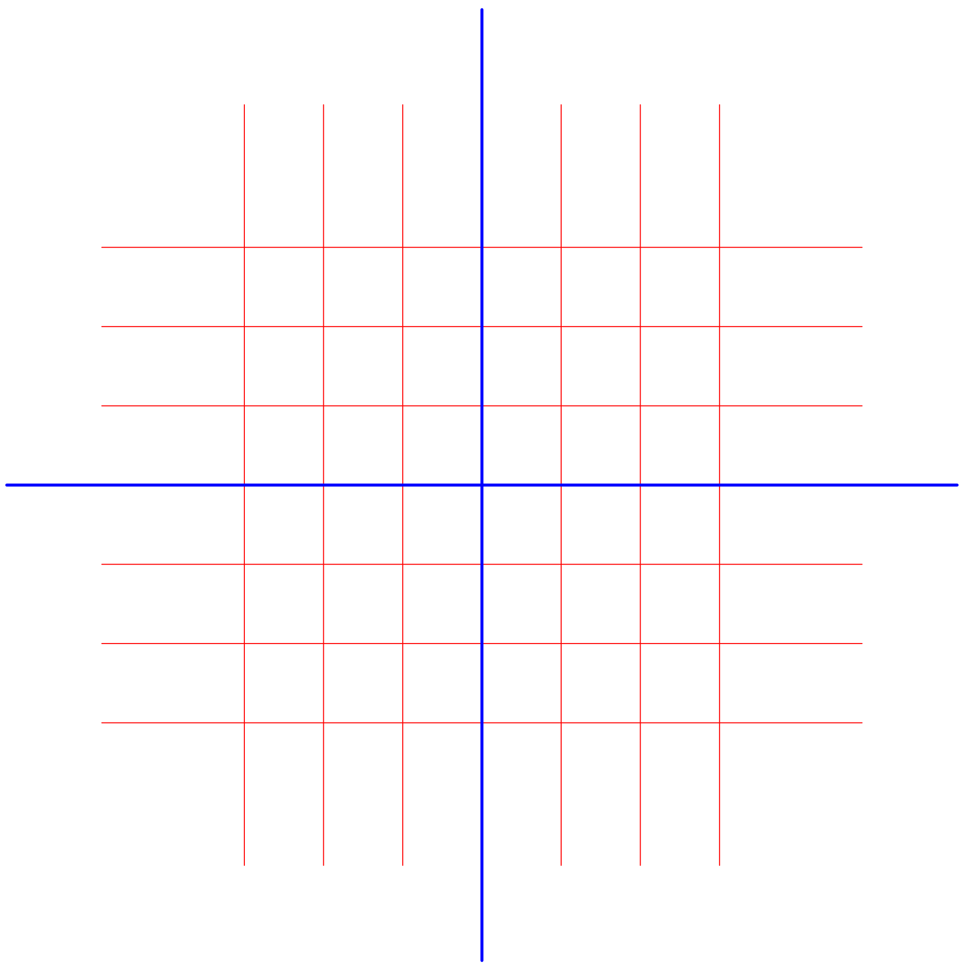
(same function as in last problem) which contains the point $(0, 0, \pi/2)$.

Space for work

Problem 7)	(10 points)

Consider the function $f(x,y) = x^2 - y^2$ of two variables.

a) Sketch in the diagram below the level curves $f(x,y) = c$ for values $c = 0, 1, 3$.
b) Find and draw the gradient vector $\nabla f(x,y)$ at the point $(2,1)$.
c) Find the directional derivative $D_u f$ of $f(x,y)$ at the point $(2,1)$ into the direction $\vec{u} = (3,4)/5$.



Space for work

Problem 8) (10 points)

A drawer of length x, width y and height z is open on the top and has the volume 32. For which dimensions are the material costs minimal?

Hint. The problem is to minimize $f(x, y, z) = xy + 2yz + 2xz$ under the constraint $g(x, y, z) = xyz = 32$.

Space for work

Problem 9) (10 points)

A stone follows a path $\vec{r}(t) = (x(t), y(t), z(t))$. It has the initial velocity $\vec{r}\,'(0) = (3, 0, 3)$ and is launched at the point $(5, 10, 14)$. It is subject to the gravitational force which means $\vec{r}\,''(t) = (0, 0, -10)$. Where will the stone hit the ground $z = 0$?

Space for work