

Name:

- Start by printing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) (20 points) No justifications are needed.

T F

The vector $\vec{v} = (1, 5, 7)$ is perpendicular to the plane $x + 5y + 7z = 100$.

T F

For any three vectors $\vec{u}, \vec{v}, \vec{w}$, the identity $\vec{u} \cdot (\vec{v} \times \vec{w}) = (\vec{w} \times \vec{v}) \cdot \vec{u}$ holds.

T F

With $\vec{i} = (1, 0, 0), \vec{j} = (0, 1, 0), \vec{k} = (0, 0, 1)$, the formula $(\vec{i} \times \vec{k}) \times (\vec{j} \times \vec{k}) = \vec{0}$ holds.

T F

The vectors $\vec{u} = (3, -2, 1)$ and $\vec{v} = (-6, 4, 2)$ are parallel.

T F

The set of points which have distance 1 from the xy -plane is a single plane parallel to the xy -plane.

T F

If $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$ are perpendicular, then the vectors \vec{u} and \vec{v} have the same length.

T F

The two vectors $(2, 3, 0)$ and $(6, -4, 5)$ are orthogonal.

T F

For any two vectors \vec{v}, \vec{w} , one has $|\vec{v} + \vec{w}|^2 = |\vec{v}|^2 + |\vec{w}|^2$.

T F

The surface $x^2 - y^2 + z^2 = 1$ is called a one-sheeted hyperboloid.

T F

The set of points which have distance 1 from the x -axis is a cylinder.

T F

If in spherical coordinates a point is given by $(\rho, \theta, \phi) = (2, \pi/2, \pi/2)$, then its Euclidean coordinates is $(x, y, z) = (0, 2, 0)$.

T F

If the distance between two lines is zero, then the two lines belong to the same plane.

T F

A surface which is given as $r = 2 + \sin(z)$ in cylindrical coordinates stays the same when we rotate it around the z axis.

T F

The length of the difference $\vec{v} - \vec{w}$ of two parallel vectors \vec{v}, \vec{w} is the difference $|\vec{v}| - |\vec{w}|$ of the lengths of the vectors.

T F

The volume of a parallelepiped spanned by $(1, 0, 0), (0, 1, 0)$ and $(1, 1, 1)$ is equal to $1/3$.

T F

The equation $x^2 - z^2 = y$ describes a hyperbolic paraboloid.

T F

In spherical coordinates, the equation $\cos(\theta) = \sin(\theta)$ is the plane $x - y = 0$.

T F

If $|\vec{v} \times \vec{w}| = 0$ then $\vec{v} = 0$ or $\vec{w} = 0$.

T F

The vector projection of the vector $(1, 1, 1)$ onto the vector $(0, 2, 0)$ is $(0, 1, 0)$.

T F

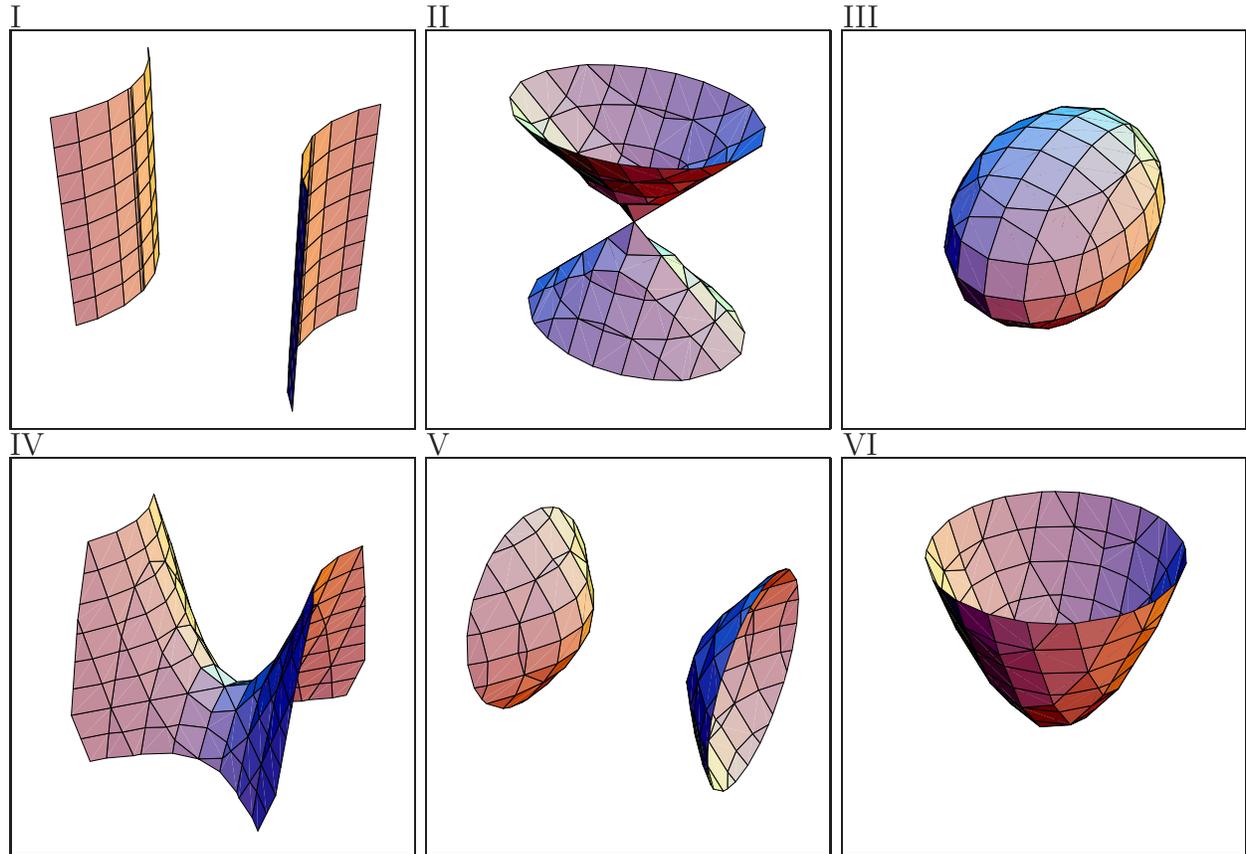
To every point on the unit sphere corresponds exactly one spherical coordinate (ρ, θ, ϕ) with $\rho = 1, 0 \leq \theta < 2\pi, 0 \leq \phi \leq \pi$.

Total

Space for work

Problem 2) (10 points)

Match the equations with the surfaces.

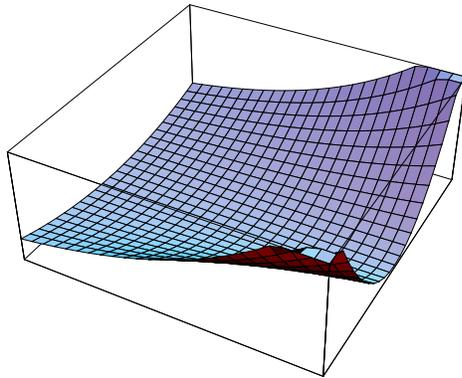


Enter I,II,III,IV,V,VI here	Equation
	$x^2 - y^2 - z^2 = 1$
	$x^2 + 2y^2 = z^2$
	$2x^2 + y^2 + 2z^2 = 1$
	$x^2 - y^2 = 5$
	$x^2 - y^2 - z = 1$
	$x^2 + y^2 - z = 1$

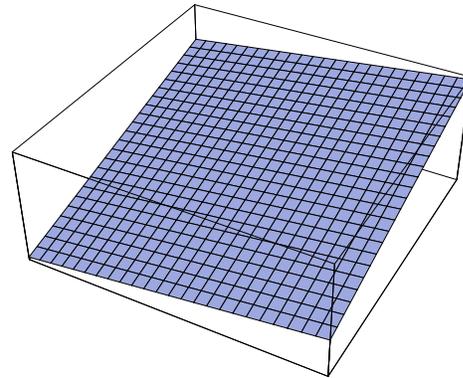
Space for work

Problem 3) (10 points)

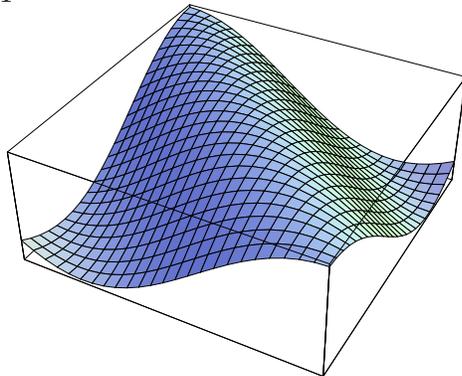
Match the equation with their graphs. No justifications are needed.



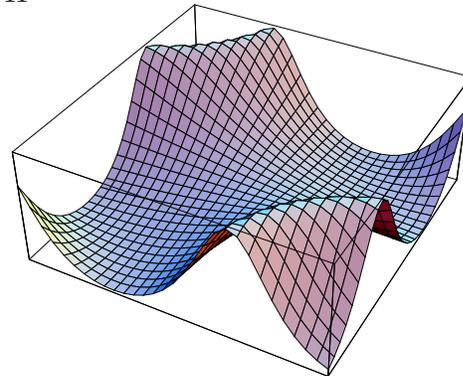
I



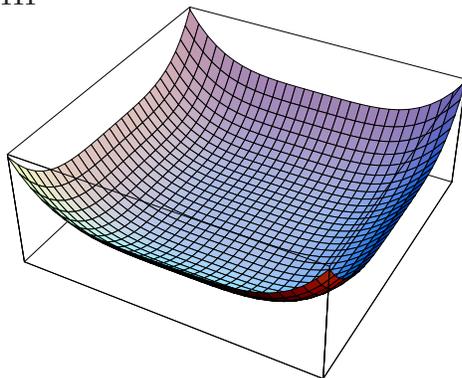
II



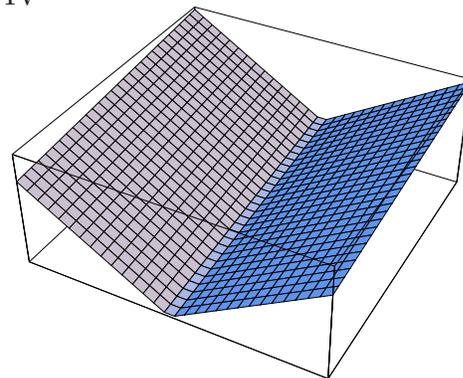
III



IV



V



VI

Enter I,II,III,IV,V or VI here	Equation
	$z = x + y$
	$z = y^2 e^x$
	$z = \cos(x + y)$
	$z = x + y$
	$z = x/(2 + \sin(xy))$
	$z = x^4 + y^4$

Space for work

Problem 4) (10 points)

Compute:

a) $(4, 5, 1) \cdot (1, -1, 1)$

b) $(1, 1, 3) \times (1, 1, 1)$

c) $(2, 1, 3) \cdot ((3, 4, 5) \times (1, 1, 3))$.

d) $\vec{\text{proj}}_{(1,0,0)}(7, 3, 2)$

e) $\text{comp}_{(0,1,0)}(7, 3, 2)$

Space for work

Problem 5) (10 points)

Find the distance between the point $P = (1, 0, -1)$ and the line which contains the points $A = (1, 1, 1)$ and $B = (0, 2, 1)$.

To do so:

- a) (4 points) Find first a parametrization $\vec{r}(t) = Q + t\vec{v}$ of the line.
- b) (6 points) Now find the distance.

Space for work

Problem 6) (10 points)

a) (4 points) Find a parameterization of the line of intersection of the planes $3x - 2y + z = 7$ and $x + 2y + 3z = -3$.

Hint. Use the fact that the line goes through the point $P = (1, -2, 0)$.

b) (3 points) Find the symmetric equations

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

representing that line.

c) (3 points) Give a formula for the angle α between the two planes. You do not have to evaluate the result numerically and leaving square roots is ok.

Hint. The angle between two planes is defined as the angle between the two normal vectors of the planes.

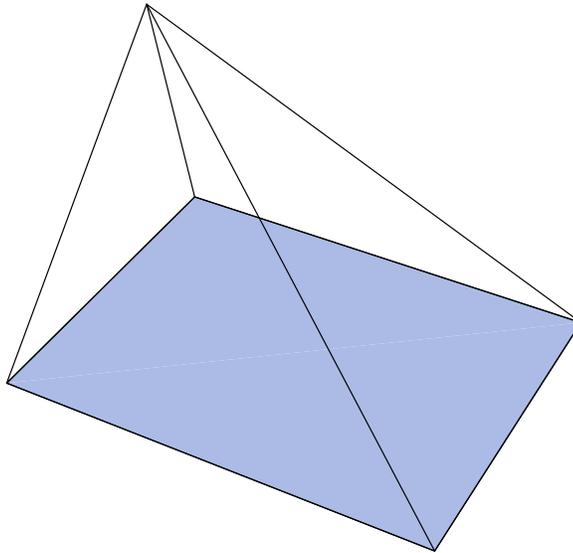
Space for work

Problem 7) (10 points)

a) (5 points) Find the area of the parallelogram $PQSR$ with corners

$$P = (0, 0, 0), Q = (1, 1, 1), R = (1, 1, 0), S = (2, 2, 1).$$

b) (5 points) Find the volume of the pyramid which has as the base the parallelogram $PQRS$ and has a fifth vertex at $T = (3, 4, 3)$.



Space for work

Problem 8) (10 points)

In this problem, it is enough to describe the surface with words.

- a) (3) Identify the surface whose equation is given in spherical coordinates as $\phi = \pi/6$.
- b) (3) Identify the surface whose equation is given in spherical coordinates as $\theta = \pi/2$.
- c) (2) Identify the surface, whose equation is given in cylindrical coordinates by $z^2 = r$.
- d) (2) Identify the surface, whose equation is given in cylindrical coordinates as $r \cos(\theta) = 1$

Space for work

Problem 9) (10 points)

Remember that a parameterization of a surface describes the points (x, y, z) of the surface in the form $\vec{r}(u, v) = (x, y, z) = (x(u, v), y(u, v), z(u, v))$. What surfaces do the following parameterizations represent? Find in each case an implicit equation of the form $g(x, y, z) = c$ which is equivalent.

a) (3) $\vec{r}(u, v) = (\cos(u), \sin(u), v)$

b) (3) $\vec{r}(u, v) = (u + v, v - u, u + 2v)$

c) (2) $\vec{r}(u, v) = (v \cos(u), v \sin(u), v)$

d) (2) $\vec{r}(u, v) = (\cos(u) \sin(v), \sin(u) \sin(v), \cos(v))$.

Space for work