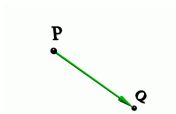
DISTANCES **Maths21a, O. Knill**

DISTANCE POINT-POINT (3D). If P and Q are two points, then

$$d(P,Q) = |\vec{PQ}|$$

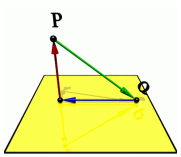
is the distance between P and Q.



DISTANCE POINT-PLANE (3D). If P is a point in space and $\Sigma : \vec{n} \cdot \vec{x} = d$ is a plane containing a point Q, then

$$d(P,\Sigma) = |(\vec{PQ}) \cdot \vec{n}|/|\vec{n}|$$

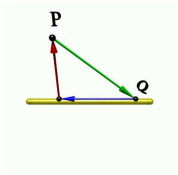
is the distance between P and the plane.



DISTANCE POINT-LINE (3D). If P is a point in space and L is the line $\vec{r}(t) = Q + t\vec{u}$, then

$$d(P,L) = |(\vec{PQ}) \times \vec{u}|/|\vec{u}|$$

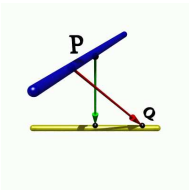
is the distance between P and the line L.



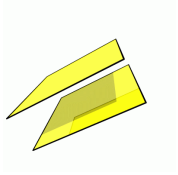
DISTANCE LINE-LINE (3D). L is the line $\vec{r}(t) = Q + t\vec{u}$ and M is the line $\vec{s}(t) = P + t\vec{v}$, then

$$d(L,M) = |(\vec{PQ}) \cdot (\vec{u} \times \vec{v})|/|\vec{u} \times \vec{v}|$$

is the distance between the two lines L and M.



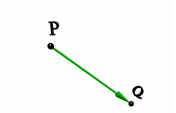
DISTANCE PLANE-PLANE (3D). If $\vec{n} \cdot \vec{x} = d$ and $\vec{n} \cdot \vec{x} = e$ are two parallel planes, then their distance is $(e-d)/|\vec{n}|$. Nonparallel planes have distance 0.



EXAMPLES

DISTANCE POINT-POINT (3D). $P = (-5,2,4)$ and $Q = (-2,2,0)$ are two points, then

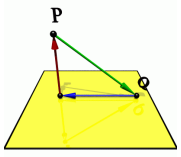
$$d(P,Q) = |\vec{PQ}| = \sqrt{(-5+2)^2 + (2-2)^2 + (0-4)^2} = 5$$



DISTANCE POINT-PLANE (3D). $P = (7,1,4)$ is a point and $\Sigma : 2x+4y+5z = 9$ is a plane which contains the point $Q = (0,1,1)$. Then

$$d(P,\Sigma) = |(-7,0,-3) \cdot (2,4,5)|/|\sqrt{45}| = 29/\sqrt{45}$$

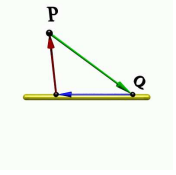
is the distance between P and Σ.



DISTANCE POINT-LINE (3D). $P = (2,3,1)$ is a point in space and L is the line $\vec{r}(t) = (1,1,2) + t(5,0,1)$. Then

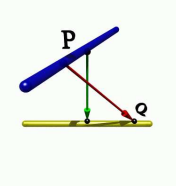
$$d(P,L) = |(-1,-2,1) \times (5,0,1)|/\sqrt{26} = |(-2,6,10)|/\sqrt{26} = \sqrt{140}/\sqrt{26}$$

is the distance between P and L.



DISTANCE LINE-LINE (3D). L is the line $\vec{r}(t) = (2,1,4) + t(-1,1,0)$ and M is the line $\vec{s}(t) = (-1,0,2) + t(5,1,2)$. The cross product of $(-1,1,0)$ and $(5,1,2)$ is $(2,2,-6)$. The distance between these two lines is

$$d(L,M) = |(3,1,2) \cdot (2,2,-6)|/\sqrt{44} = 4/\sqrt{44}\,.$$



DISTANCE PLANE-PLANE (3D). $5x + 4y + 3z = 8$ and $5x + 4y + 3z = 1$ are two parallel planes. Their distance is $7/\sqrt{50}$.