

This is part 1 (of 3) of the homework which is due July 12 at the beginning of class.

SUMMARY.

- $\vec{r}(t, s) = P + t\vec{v} + s\vec{w}$ **parametric equation** for a **plane**. $P = (x_0, y_0, z_0)$ a point \vec{v}, \vec{w} are vectors.
- $ax + by + cz = d$, **implicit equation** for a **plane**.
- $\vec{r}(t) = P + t\vec{v}$ **parametric equation** for a **line**, P a point, \vec{v} is a vector.
- $\frac{(x-x_0)}{a} = \frac{(y-y_0)}{b} = \frac{(z-z_0)}{c}$ **symmetric equation** for a **line**.
- Distance Point-Point $d(P, Q) = |\vec{PQ}|$.
- Distance Point-Plane $d(P, \Sigma) = |(\vec{PQ}) \cdot \vec{n}|/|\vec{n}|$.
- Distance Point-Line $d(P, L) = |(\vec{PQ}) \times \vec{u}|/|\vec{u}|$.
- Distance Line-Line $d(L, M) = |(\vec{PQ}) \cdot (\vec{u} \times \vec{v})|/|\vec{u} \times \vec{v}|$.

Homework Problems

- 1) (4 points) Find the equation for the plane which contains the three points $P = (1, 2, 3), Q = (3, 4, 4)$ and $R = (1, 1, 2)$.

Solution:

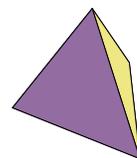
A normal vector $\vec{n} = (1, -2, 2) = (a, b, c)$ of the plane $ax + by + cz = d$ is obtained as the cross product of $P - Q$ and $R - Q$. With $d = \vec{n} \cdot P = 3$, we have the equation $x - 2y + 2z = 3$.

- 2) (4 points)
 a) (3) Find the distance between the point $(2, -1, 2)$ and the plane $4x - 2y + z = 2$.
 b) (1) If no absolute value is taken in the distance formula, what does the sign of the result say?

Solution:

- a) The point $Q = (0, 0, 2)$ is on the plane. The scalar projection of $P - Q = (2, -1, 0)$ onto the normal vector $(4, -2, 1)$ of the plane is $10/\sqrt{21}$.
 b) If $(P - Q) \cdot \vec{n}/|\vec{n}|$ is positive, then the point P is on the side into which the normal vector points.

- 3) (4 points) A regular tetrahedron has vertices at the points $P_1 = (0, 0, 3), P_2 = (0, \sqrt{8}, -1), P_3 = (-\sqrt{6}, -\sqrt{2}, -1)$ and $P_4 = (\sqrt{6}, -\sqrt{2}, -1)$. Find the distance between two edges which do not intersect.

**Solution:**

The vector $\vec{v} = (P_2 - P_1) = (0, 2\sqrt{2}, -4)$ is parallel to the first edge, the vector $\vec{w} = (P_4 - P_3) = (2\sqrt{6}, 0, 0)$ is parallel to the second edge. The cross product of \vec{v} and \vec{w} is $\vec{n} = (0, -8\sqrt{6}, -8\sqrt{3})$. The distance between the two edges is the scalar projection of $P_3 - P_1$ onto \vec{n} . It is $(P_3 - P_1) \cdot \vec{n}/|\vec{n}| = 2\sqrt{3}$.

- 4) (4 points) a) Find a parametric equation for the line through the point $P = (3, 1, 2)$ that is perpendicular to the line $L : x = 1 + t, y = 1 - t, z = 2t$ and intersects this line in a point Q .

Solution:

b) The point $Q = (1, 1, 0)$ is on the line. The vector $\vec{v} = (1, -1, 2)$ parallel to the line. We have $P - Q = (2, 0, 2)$. The vector $\vec{n} = \vec{v} \times (P - Q) = (-6, -6, 0)$ is the direction from P to the normal intersection with the line. The line can be given by $\vec{r}(t) = (3 - 6t, 1 - 6t, 2)$.

- 5) (4 points) Compute the distance of P to L in the previous problem and verify that it is equal to $d(P, Q)$.

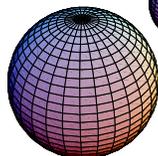
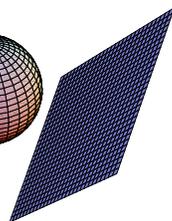
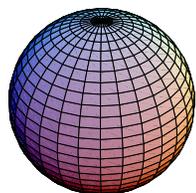
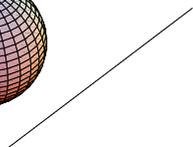
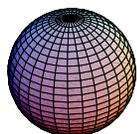
Solution:

The solution is $\sqrt{2}$. You can get the answer using the distance formula $|(P - Q) \times v|/|v|$.

Challenge Problems

(Solutions to these problems are **not** turned in with the homework.)

- 1) Find distance formulas between
 a) A point and a sphere.
 b) A plane and a sphere.
 c) Two spheres.
 d) A line and a sphere.



- 2) How does one describe a three dimensional "hyper-plane" in four dimensional space? Find a parametric description and an implicit description.
- 3) Can you find a line and a two dimensional plane in \mathbf{R}^4 which are not parallel and which do not intersect? How would you compute the distance between a line and a two dimensional plane in \mathbf{R}^4 ?