Name:

- Start by printing your name in the above box.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) (20 points) No justifications needed.

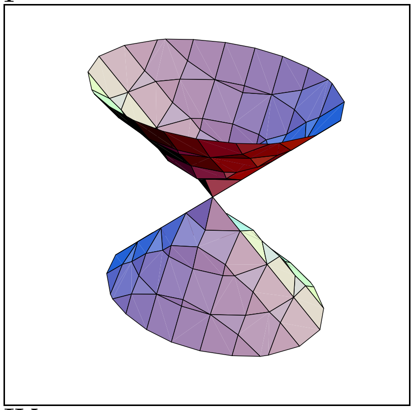
T	F	The vector connecting the point $P=(1,4,3)$ with the point $Q=(2,2,2)$ is parallel to the vector $\vec{v}=(-1,2,1)$.
T	**F**	For any three vectors the identity $(\vec{v}\times\vec{w})\cdot\vec{u}=(\vec{w}\times\vec{v})\cdot\vec{u}$ holds.
T	F	For any vector $\vec{v}$ one has $\vec{v}\times(7\vec{v})=\vec{0}$.
T	**F**	The set of points which have distance 1 from the xy-plane is a single plane parallel to the xy-plane.
T	**F**	The vectors $(3,-2,1)$ and $(-6,4,2)$ are parallel.
T	F	If $\vec{u}+\vec{v}$ and $\vec{u}-\vec{v}$ are perpendicular, then the vectors $\vec{u}$ and $\vec{v}$ have the same length.
T	F	The vectors $(2,3)$ and $(6,-4)$ are orthogonal.
T	**F**	For any two vectors $\vec{v},\vec{w}$ one has $\lvert\vec{v}+\vec{w}\rvert^2=\lvert\vec{v}\rvert^2+\lvert\vec{w}\rvert^2$.
T	F	The surface $x^2-y^2+z^2=1$ is a one-sheeted hyperboloid.
T	F	The set of points which have distance 1 from the x-axis is a cylinder.
T	F	If in spherical coordinates a point is given by $(\rho,\theta,\phi)=(2,\pi/2,\pi/2)$, then its rectangular coordinates are $(x,y,z)=(0,2,0)$.
T	**F**	The equation $r=3$ in cylindrical coordinates is a sphere.
T	F	A surface which is given as $r=2+\sin(z)$ in cylindrical coordinates stays the same when we rotate it around the z axis.
T	**F**	The length of the difference $\vec{v}-\vec{w}$ of two parallel vectors is the difference $\lvert\vec{v}\rvert-\lvert\vec{w}\rvert$ of the lengths of the vectors.
T	**F**	The volume of a parallelepiped spanned by $(1,0,0),(0,1,0)$ and $(1,1,1)$ is equal to $1/3$.
T	**F**	The equation $x^2-z^2=y$ describes a paraboloid.
T	F	In spherical coordinates the equation $\cos(\theta)=\sin(\theta)$ defines the plane $x-y=0$.
T	**F**	If $\lvert\vec{v}\times\vec{w}\rvert=0$ then $\vec{v}=0$ or $\vec{w}=0$.
T	F	The projection of the vector $(1,1,1)$ onto the vector $(0,2,0)$ is $(0,1,0)$.
T	**F**	The scalar projection of a vector $\vec{v}$ onto a vector $\vec{w}$ is always equal to the scalar projection of $\vec{w}$ onto $\vec{v}$.

	Total

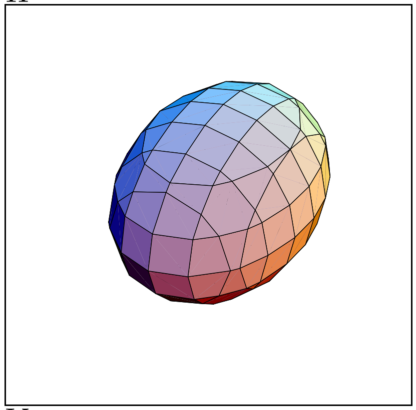
Problem 2) (10 points)

Match the equations with the surfaces.

I



II



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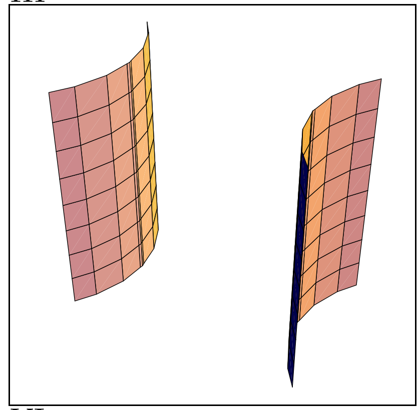

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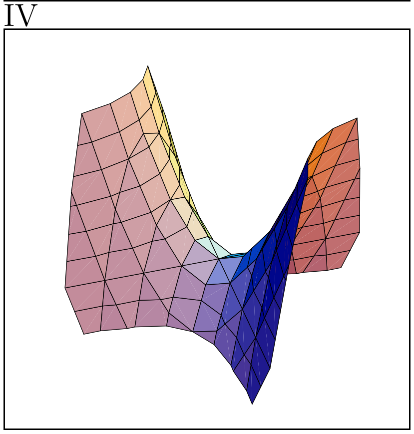

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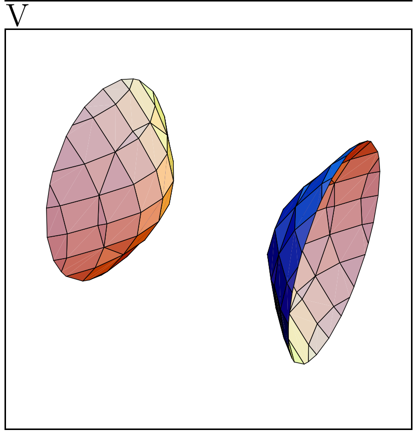
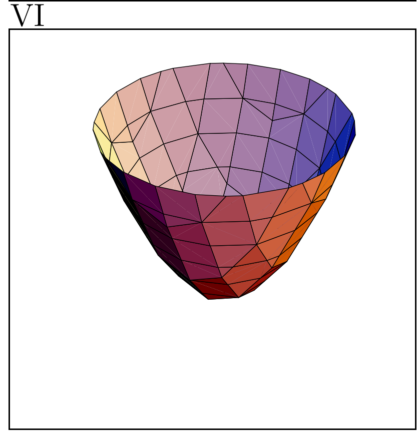


VI



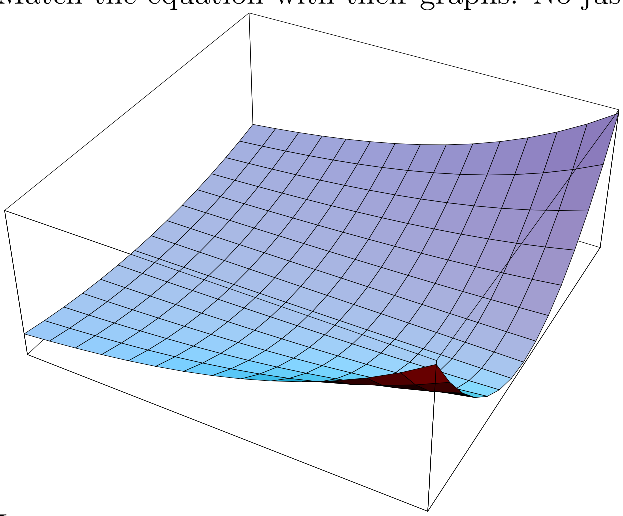
Enter I,II,III,IV,V,VI here	Equation
	$x^2 - y^2 - z^2 = 1$
	$x^2 + 2y^2 = z^2$
	$2x^2 + y^2 + 2z^2 = 1$
	$x^2 - y^2 = 5$
	$x^2 - y^2 - z = 1$
	$x^2 + y^2 - z = 1$

Solution:

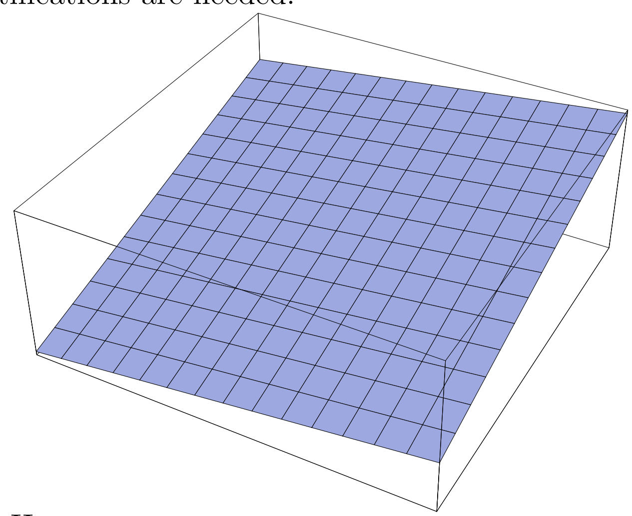
Enter I,II,III,IV,V,VI here	Equation
V	$x^2 - y^2 - z^2 = 1$
I	$x^2 + 2y^2 = z^2$
II	$2x^2 + y^2 + 2z^2 = 1$
III	$x^2 - y^2 = 5$
IV	$x^2 - y^2 - z = 1$
VI	$x^2 + y^2 - z = 1$

Problem 3) (10 points)

Match the equation with their graphs. No justifications are needed.



I



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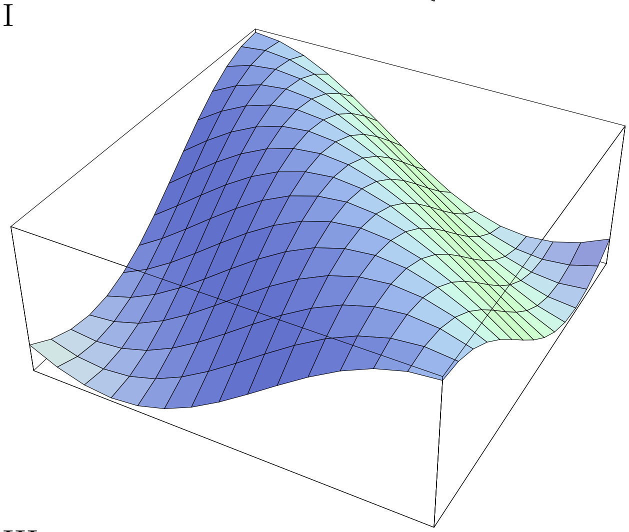
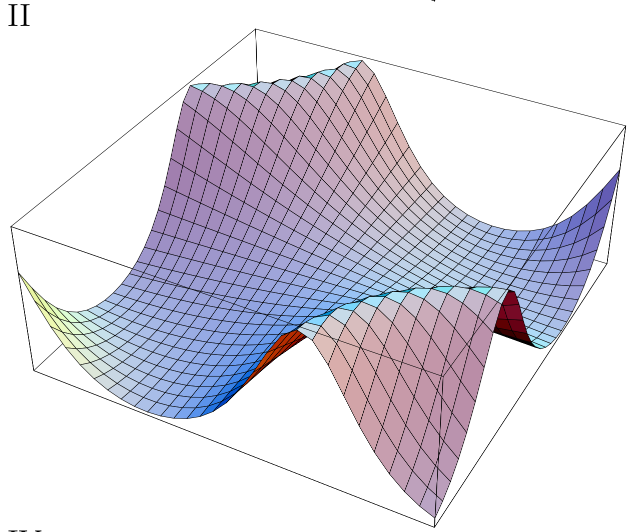


III



IV

Enter I,II,III,IV here	Equation
	$z = y^2 e^x$
	$z = \cos(x+y)$
	$z = x + y$
	$z = x/(2 + \sin(xy))$

Solution:

Enter I,II,III,IV here	Equation
I	$z = y^2 e^x$
III	$z = \cos(x+y)$
II	$z = x + y$
IV	$z = x/(2 + \sin(xy))$

Problem 4) (10 points)

Compute:

a) $(3, 2, 1) - (1, 3, 5)$
b) $|(\sqrt{7}, 3, 3)|$
c) $(4, 5, 1) \cdot (1, 1, 1)$
d) $(1, 1, 3) \times (1, 1, 1)$
e) $(2, 1, 3) \cdot ((3, 4, 5) \times (1, 1, 3))$.

Solution:
a) $(2, -1, -4)$.
b) 5.
c) 10.
d) $(-2, 2, 0)$.
e) $((3, 4, 5) \times (2, 1, 3)) = (7, -4, -1)$ and the result is 7.

Problem 5) (10 points)

Find the distance between the point $P = (1, 0, -1)$ and the plane which contains the points $A = (1, 1, 1)$ and $B = (0, 2, 1)$ and $C = (1, 2, 2)$.
To do so:

a) Find the equation $ax + by + cz = d$ of the plane.

Solution:
$n = ((0,2,1) - (1,1,1)) \times ((1,2,2) - (1,1,1)) = (1,1,-1)$ is normal to the plane. Therefore $x + y - z = d = 1$. The constant d was obtained by pluggin in the coordinates of one of the points on the plane.

b) Find the distance.

Solution:
Project the vector $\vec{PA}$ onto the vector $\vec{n}$ to obtain $d = 1/\sqrt{3}$.

Problem 6) (10 points)

a) (6 points) Find a parameterization of the line of intersection of the planes $3x - 2y + z = 7$ and $x + 2y + 3z = -3$.

Hint. Use the fact that the line goes through the point $P = (1, -2, 0)$.

b) (4 points) Find the symmetric equations

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

representing that line.

Solution:
a) The line of intersection has the direction $(3,-2,1) \times (1,2,3) = 8(-1,-1,1)$. The parameterization is $\boxed{\vec{r}(t) = (1,-2,0) + t(-1,-1,1)}$.
b) If a line contains the point (x_0, y_0, z_0) and a vector (a, b, c), then the symmetric equation is $(x - x_0)/a = (y - y_0)/b = (z - z_0)/c$. In our case, where $(x_0, y_0, z_0) = (1, -2, 0)$ and $(a, b, c) = (-1, -1, 1)$, the symmetric equations are $\boxed{x - 1 = y + 2 = -z}$.

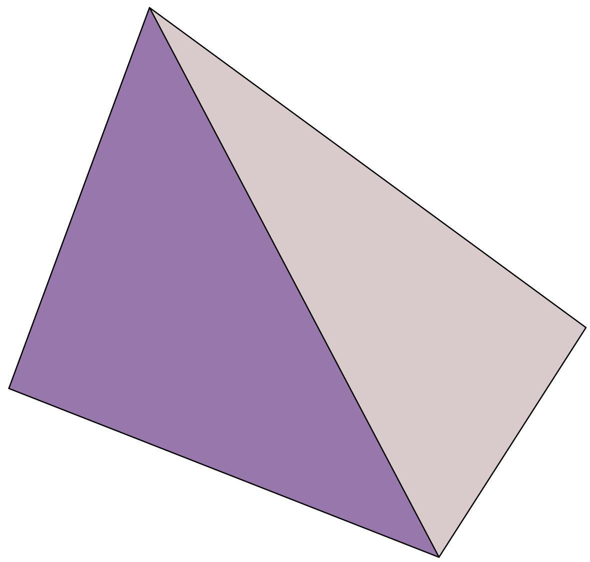
Problem 7) (10 points)

Find the volume of the pyramid which has as the base the parallelogram with vertices

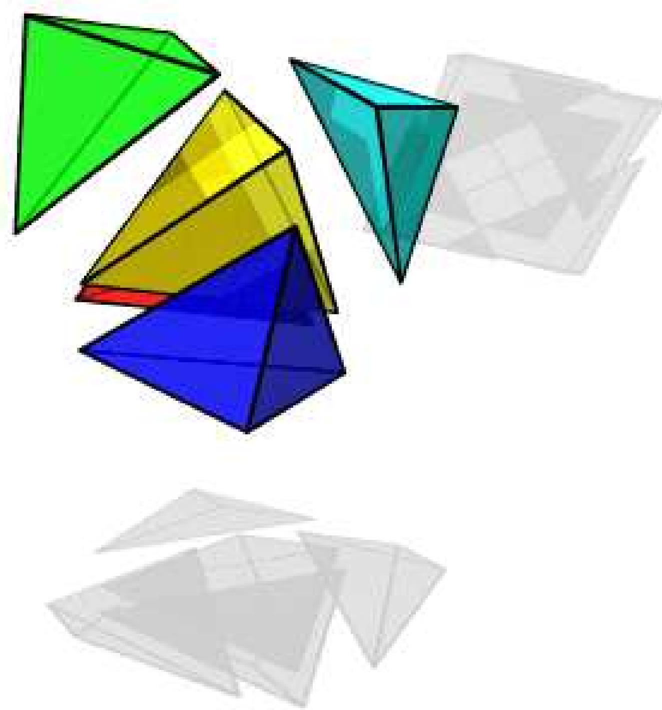
$$P = (0, 0, 0), Q = (1, 1, 1), R = (1, 1, 0), S = (2, 2, 1)$$

and has the fifth vertex at $T = (3, 4, 3)$.

Hint. You can use that the pyramid has 1/3 of the volume of the parallelepiped.



Solution:
The volume of the parallelepiped spanned by the vectors $\vec{u} = (1,1,1), \vec{v} = (1,1,0)$ and $\vec{w} = (2,2,1)$ is 1. The pyramid has volume 1/3 because one can chop the pyramid into two tetraedra of the same volume and each tetrahedron has volume 1/6. One can take 4 tetrahedra spanned by u, v, w as well as one of double volume spanned by $(u+v, u+w, v+w)$ to build the parallelepiped. The volume of the pyramid is therefore $\boxed{1/3}$.
By the way: the following picture is an example to reason that a tetrahedron spanned by vectors u, v, w has a volume 1/6'th of the volume of the parallelepiped:



An other (more laborous but more straightforward) solution of the problem is to compute the distance $h = |w \cdot (u \times v)|/|w|$ of T to the plane containing the points P, Q, S, R and using the volume formula $V = Ah/3$, where $A = |u \times v|$ is the area of the parallelogram.

Problem 8) (10 points)

a) (3) Identify the surface whose equation is given in spherical coordinates as $\theta = \pi/2$.

b) (3) Identify the surface whose equation is given in spherical coordinates as $\phi = \pi/6$.

c) (2) Identify the surface, whose equation is given in cylindrical coordinates by $z^2 = r$.

d) (2) Identify the surface, whose equation is given in cylindrical coordinates as $r\cos(\theta) = 1$

Solution:
a) A half plane contained in the plane $x = 0$ and containing the positive y axes. This example was covered in lecture.
b) A half cone. $x^2 + y^2 = z^2$ with $z \geq 0$.
c) This surface appeared in the homework. It is a surface of revolution.
d) This is the plane $x = 1$. Just note that $r\cos(\theta) = x$ in cylindrical coordiantes.

Problem 9) (10 points)

Remember that a parameterization of a surface describes the points (x, y, z) of the surface in the form $(x, y, z) = (x(u, v), y(u, v), z(u, v))$. What surfaces do the following parameterizations represent? Give a name to each surface if you know it and find in each case an implicit equation of the form $g(x, y, z) = c$ which is equivalent.

a) (3) $\vec{r}(u, v) = (\cos(u), \sin(u), v)$

b) (3) $\vec{r}(u, v) = (u + v, v - u, u + 2v)$

c) (2) $\vec{r}(u, v) = (v\cos(u), v\sin(u), v)$

d) (2) $\vec{r}(u, v) = (\cos(u)\sin(v), \sin(u)\sin(v), \cos(v))$.

Solution:
a) The cylinder $x^2 + y^2 = 1$.
b) A plane containing the vectors $(1, -1, 1)$ and $(1, 1, 2)$. We have $-3x - y + 2z = 0$.
c) A cone $x^2 + y^2 = z^2$.
d) The unit sphere $x^2 + y^2 + z^2 = 1$.