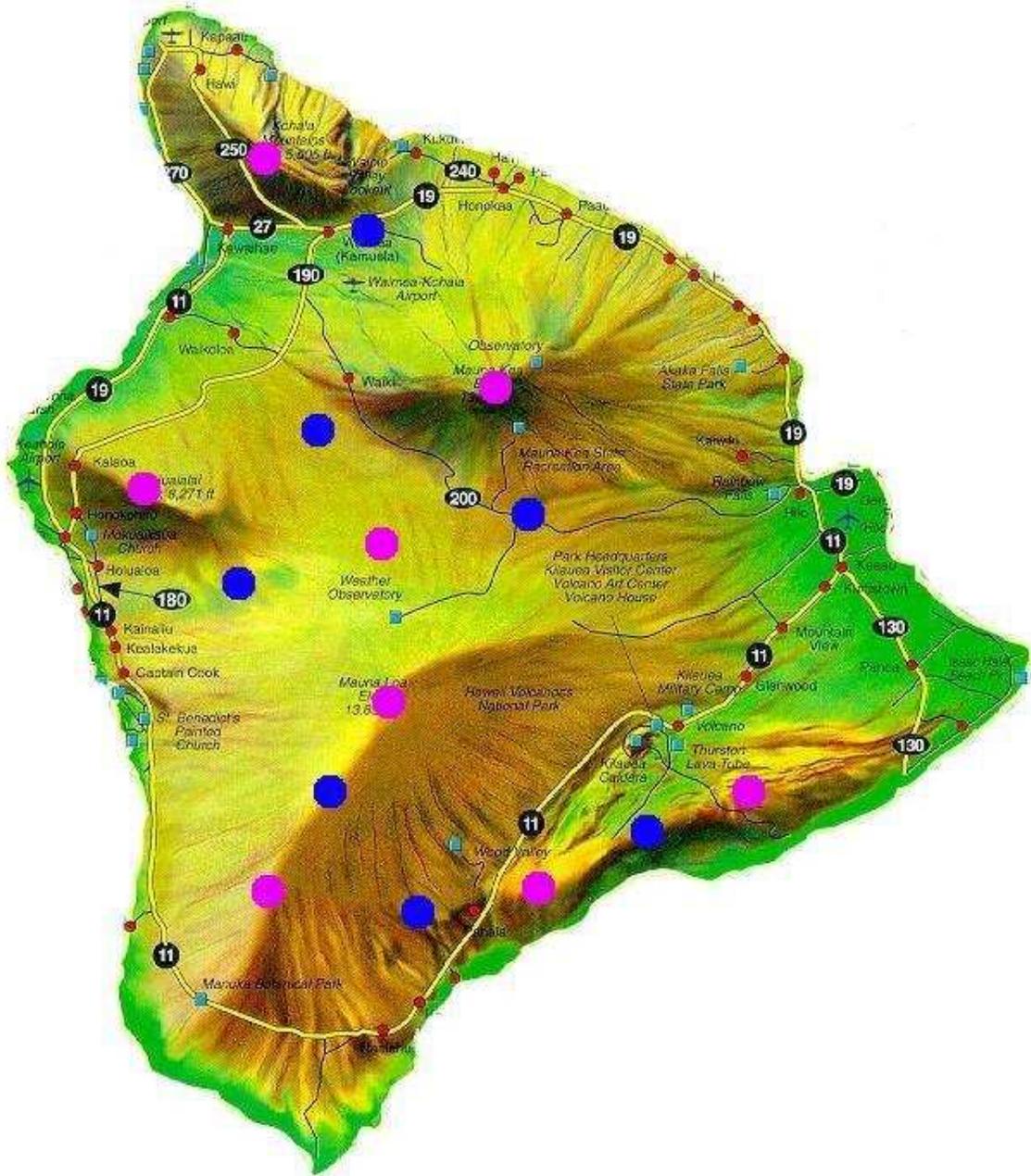


Find all the critical points of $f(x, y) = x^5y + xy^5 + xy$ and classify them.

- 1) Find the gradient $\nabla f(x, y)$.
- 2) Find the critical points $\nabla f(x, y) = (0, 0)$.
- 3) Find the discriminant D at each critical point.
- 4) If $D > 0$, find the sign of H_{11} .
- 5) Classify the critical points.

You are on a circular island surrounded by sea. You count A mountain peaks, B sinks and C mountain passes. You observe that $A + B - C = 1$. Can you see why this is true for any island?



The example shows the Hawaii main island. On a rough scale, we have $A = 7$ mountain peaks, $B = 1$ sinks and $C = 7$ mountain passes.

- 1) Explore the rule with simple examples.
- 2) Look what happens if you deform a given island to an island, with only one mountain and no sink and no mountain pass.
- 3) Can you see what happens during the deformation?