

This is part 1 (of 3) of the homework which is due July 13 at the beginning of class.

**SUMMARY.**

- $\vec{r}(t, s) = P + t\vec{v} + s\vec{w}$  **parametric equation** for a **plane**.  $P = (x_0, y_0, z_0)$  a point  $\vec{v}, \vec{w}$  are vectors.
- $ax + by + cz = d$ , **implicit equation** for a **plane**.
- $\vec{r}(t) = P + t\vec{v}$  **parametric equation** for a line,  $P$  a point,  $\vec{v}$  is a vector.
- $\frac{(x-x_0)}{a} = \frac{(y-y_0)}{b} = \frac{(z-z_0)}{c}$  **symmetric equation** for a line.
- Distance Point-Point  $d(P, Q) = |\vec{PQ}|$ .
- Distance Point-Plane  $d(P, \Sigma) = |(\vec{PQ}) \cdot \vec{n}|/|\vec{n}|$ .
- Distance Point-Line  $d(P, L) = |(\vec{PQ}) \times \vec{u}|/|\vec{u}|$ .
- Distance Line-Line  $d(L, M) = |(\vec{PQ}) \cdot (\vec{u} \times \vec{v})|/|\vec{u} \times \vec{v}|$ .

## Homework Problems

- 1) (4 points) Find the equation for the plane which contains the three points  $P = (1, 2, 3), Q = (3, 4, 4)$  and  $R = (1, 1, 2)$ .

**Solution:**

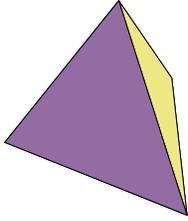
A normal vector  $\vec{n} = (1, -2, 2) = (a, b, c)$  of the plane  $ax + by + cz = d$  is obtained as the cross product of  $P - Q$  and  $R - Q$ . With  $d = \vec{n} \cdot P = 3$ , we have the equation  $x - 2y + 2z = 3$ .

- 2) (4 points)
- a) (3) Find the distance between the point  $(2, -1, 2)$  and the plane  $4x - 2y + z = 2$ .
- b) (1) If no absolute value is taken in the distance formula, what does the sign of the result say?

**Solution:**

- a) The point  $Q = (0, 0, 2)$  is on the plane. The scalar projection of  $P - Q = (2, -1, 0)$  onto the normal vector  $(4, -2, 1)$  of the plane is  $10/\sqrt{21}$ .
- b) If  $(P - Q) \cdot \vec{n}/|\vec{n}|$  is positive, then the point  $P$  is on the side into which the normal vector points.

- 3) (4 points) A regular tetrahedron has vertices at the points  $P_1 = (0, 0, 3), P_2 = (0, \sqrt{8}, -1), P_3 = (-\sqrt{6}, -\sqrt{2}, -1)$  and  $P_4 = (\sqrt{6}, -\sqrt{2}, -1)$ . Find the distance between two edges which do not intersect.



**Solution:**

The vector  $\vec{v} = (P_2 - P_1) = (0, 2\sqrt{2}, -4)$  is parallel to the first edge, the vector  $\vec{w} = (P_4 - P_3) = (2\sqrt{6}, 0, 0)$  is parallel to the second edge. The cross product of  $\vec{v}$  and  $\vec{w}$  is  $\vec{n} = (0, -8\sqrt{6}, -8\sqrt{3})$ . The distance between the two edges is the scalar projection of  $P_3 - P_1$  onto  $\vec{n}$ . It is  $(P_3 - P_1) \cdot \vec{n} / |\vec{n}| = 2\sqrt{3}$ .

- 4) (4 points) a) Find a parametric equation for the line through the point  $P = (3, 1, 2)$  that is perpendicular to the line  $L : x = 1 + t, y = 1 - t, z = 2t$  and intersects this line in a point  $Q$ .

**Solution:**

b) The point  $Q = (1, 1, 0)$  is on the line. The vector  $\vec{v} = (1, -1, 2)$  parallel to the line. We have  $P - Q = (2, 0, 2)$ . The vector  $\vec{n} = \vec{v} \times (\vec{v} \times (P - Q)) = (-6, -6, 0)$  is the direction from  $P$  to the normal intersection with the line. The line can be given by  $\vec{r}(t) = (3 - 6t, 1 - 6t, 2)$ .

- 5) (4 points) Compute the distance of  $P$  to  $L$  in the previous problem and verify that it is equal to  $d(P, Q)$ .

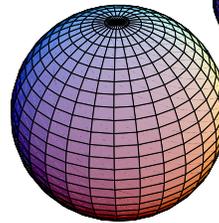
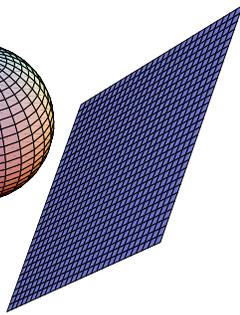
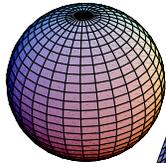
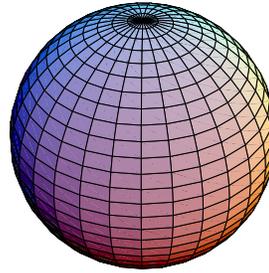
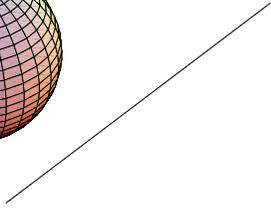
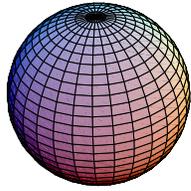
**Solution:**

The solution is  $\sqrt{2}$ . You can get the answer using the distance formula  $|(P - Q) \times v| / |v|$ .

## Challenge Problems

(Solutions to these problems are **not** turned in with the homework.)

- 1) Find distance formulas between
  - a) A point and a sphere.
  - a) A plane and a sphere.
  - c) Two spheres.
  - d) A line and a sphere.



- 2) How does one describe a three dimensional "hyperplane" in four dimensional space? Find a parametric description and an implicit description.
- 3) Can you find a line and a two dimensional plane in  $\mathbf{R}^4$  which are not parallel and which do not intersect? How would you compute the distance between a line and a two dimensional plane in  $\mathbf{R}^4$ ?