SURFACES OF REVOLUTION. Consider a positive function $g(x)$ on an interval $[a, b]$ on the x axes and and rotate the graph around the x-axes. We obtain a **surface of revolution** parameterized by $\boxed{(u, v) \mapsto \vec{r}(u, v) = (v, g(v)\cos(u), g(v)\sin(u))}$ on $R = [0, 2\pi] \times [a, b]$.

SURFACE AREA. We have $\vec{r}_u = (0, -g(v)\sin(u), g(v)\cos(u)), \vec{r}_v = (1, g'(v)\cos(u), g'(v)\sin(u))$ and $\vec{r}_u \times \vec{r}_v = (-g(v)g'(v), g(v)\cos(u), g(v)\sin(u)) = g(v)(-g'(v), \cos(u), \sin(u))$ which has the length $|\vec{r}_u \times \vec{r}_v| = |g(v)|\sqrt{1 + g'(v)^2}$. The surface area of such a surface of revolution is therefore $\boxed{2\pi \int_a^b |g(v)|\sqrt{1 + g'(v)^2}\, dv.}$

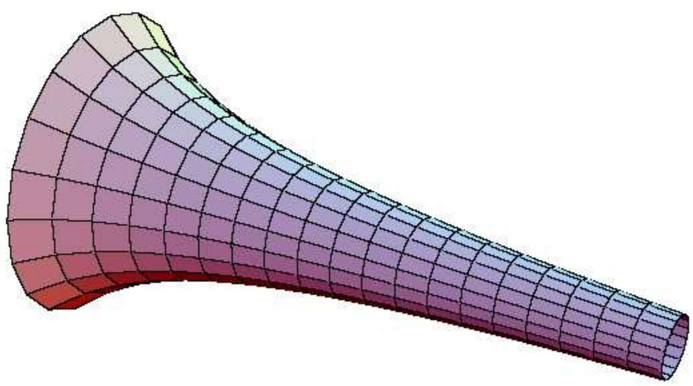
VOLUME. If we cut through the surface perpendicular to the x-axes, we obtain a disc of radius $g(x)$ and area $\pi g(x)^2$. The part of the surface between x and $x + dx$ has volume $g(x)^2 dx$. Therefore: The interior of a surface of revolution has volume $\boxed{\int_a^b \pi g(x)^2\, dx.}$

GABRIEL'S TRUMPET. The surface of revolution defined by $g(x) = 1/x$ on the interval $[1, \infty)$ is called **Gabriel's trumpet**.

Volume: The volume is $\boxed{}$

Surface area: Fill in the rest: The surface area is

$$\boxed{} \geq 2\pi \int_1^\infty \frac{1}{x}\, dx = 2\pi \log(x)|_1^\infty = \infty.$$



We conclude: the Gabriel trumpet is a surface of finite volume but with infinite surface area! You can fill the trumpet with a finite amount of paint, but this paint does not suffice to cover the surface of the trumpet!