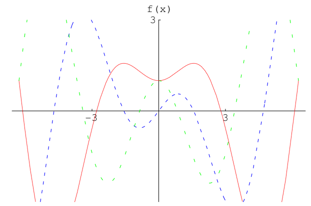
CURVES AS GRAPHS. If $f(x)$ is a function of one variable, then $\{(x, f(x))\}$ is a **graph**. Graphs are examples of curves in the plane but they include a rather narrow class of curves.

EXAMPLE. Let $f(x) = \cos(x) + x\sin(x)$ defined on $[-2\pi, 2\pi]$. The graph as well as the graph of f' and f'' is shown in the picture to the right. The graph can be traced by assigning to each x a point $(x, f(x))$ in the plane.



PARAMETRIC PLANE CURVES. If $f(t)$, $g(t)$ are functions of one variable, defined on the **parameter interval** $I = [a, b]$, then $\vec{r}(t) = (f(t), g(t))$ is a **parametric curve** in the plane. The functions $f(t), g(t)$ are called **coordinate functions** often also denoted by $(x(t), y(t))$.

SPACE CURVES. If $f(t), g(t), h(t)$ are functions, then $\vec{r}(t) = (f(t), g(t), h(t))$ is a **space curve**.

EXAMPLE 1. If $x(t) = t$, $y(t) = t^2 + 1$, we can write $y(x) = x^2 + 1$ and the curve is a **graph**.

EXAMPLE 2. If $x(t) = \cos(t), y(t) = \sin(t)$, then $\vec{r}(t)$ follows a **circle**.

If $x(t), y(t), z(t)$ are functions, then $\vec{r}(t) = (x(t), y(t), z(t))$ describes a **curve** in space.

EXAMPLE 3. If $x(t) = \cos(t), y(t) = \sin(t), z(t) = t$, then $\vec{r}(t)$ describes a **spiral**.

IDEA: Think of the **parameter** t as **time**. For every fixed t, we have a point $(x(t), y(t), z(t))$ in space. As t varies, we move along the curve.

EXAMPLE 4. If $x(t) = \cos(2t), y(t) = \sin(2t), z(t) = 2t$, then we have the same curve as in example 3 but we traverse it **faster**. The **parametrisation** changed.

EXAMPLE 5. If $x(t) = \cos(-t), y(t) = \sin(-t), z(t) = -t$, then we have the same curve as in example 3 but we traverse it in the **opposite direction**.

EXAMPLE 6. If $P = (a, b, c)$ and $Q = (u, v, w)$ are points in space, then $r(t) = (a+t(u-a), b+t(v-b), c+t(w-c))$ defined on $t \in [0, 1]$ is a **line segment** connecting P with Q.

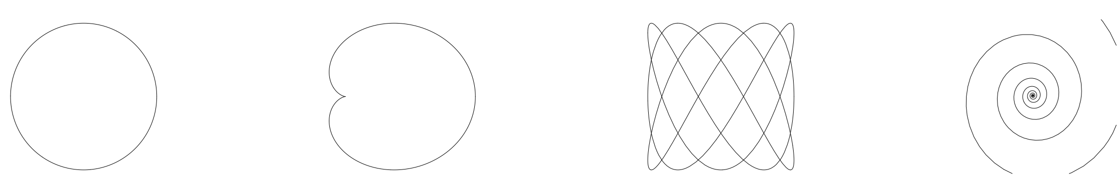
EXAMPLE 7. If $\vec{r}(u, v)$ is a parametric curve and one of the variables is fixed, then $u \mapsto r(u, v)$ is a curve called a **grid curve**. For example, if $r(\theta, \phi) = (\cos(\theta)\cos(\phi), \sin(\theta)\sin(\phi), \cos(\phi))$ is a sphere, then for fixed θ, $r(\theta, \phi)$ is called a **meridian**, a curve connecting north and south pole. For fixed ϕ, one obtains circles. The equator is a grid curve with $\phi = \pi/2$.

ELIMINATION: Sometimes it is possible to eliminate the parameter t and write the curve using equations (one equation in the plane or two equations in space).

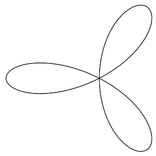
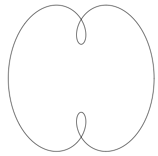
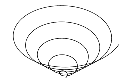
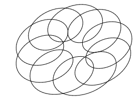
EXAMPLE: (circle) If $x(t) = \cos(t), y(t) = \sin(t)$, then $x(t)^2 + y(t)^2 = 1$.

EXAMPLE: (spiral) If $x(t) = \cos(t), y(t) = \sin(t), z(t) = t$, then $x = \cos(z), y = \sin(z)$. The spiral is the intersection of two graphs $x = \cos(z)$ and $y = \sin(z)$.

CIRCLE	HEART	LISSAJOUS	SPIRAL



$(\cos(t), 3\sin(t))$	$(1+\cos(t))(\cos(t), \sin(t))$	$(\cos(3t), \sin(5t))$	$e^{t/10}(\cos(t), \sin(t))$

TRIFOLIUM	EPICYCLE	SPRING (3D)	TORAL KNOT (3D)
			
$-\cos(3t)(\cos(t),\sin(t))$	$(\cos(t)+\cos(3t)/2, \sin(t)+\sin(3t)/2)$	$(\cos(t),\sin(t),t)$	$(\cos(t)+\cos(9t)/2, \sin(t)+\cos(9t)/2, \sin(9t)/2)$

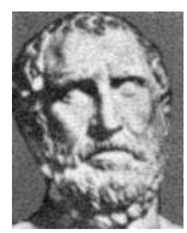
WHERE DO CURVES APPEAR? Objects like elementary particles, bodies, or quantities changing in time are described by curves. The parameter is usually the time. Examples are the motion of a star moving in a galaxy, or data changing in time like (DJIA(t),NASDAQ(t),SP500(t))

Strings or knots are curves which are important in theoretical physics. Knots are closed curves in space. Complicated **molecules** like RNA or proteines can be modeled as curves.
Computer graphics: surfaces are represented by mesh of curves.
Space time A curve in space-time describes the motion of particles.
Topology Curves are also interesting in **topology** like for space filling curves, boundaries of surfaces or knots.

DERIVATIVES. If $\vec{r}(t) = (x(t), y(t), z(t))$ is a curve, then $\boxed{\vec{r}\,'(t) = (x'(t), y'(t), z'(t)) = (\dot{x}, \dot{y}, \dot{z})}$ is called the **velocity**. The length $|\vec{r}\,'(t)|$ is the **speed**. The vector $\vec{r}\,''(t)$ is the **acceleration**. While the velocity vector is tangent to the curve the acceleration can point in any direction. The third derivative $\vec{r}\,'''$ is called the **jerk**.

EXAMPLE. If $\vec{r}(t) = (\cos(3t), \sin(2t), 2\sin(t))$, then $\vec{r}\,'(t) = (-3\sin(3t), 2\cos(2t), 2\cos(t))$, $\vec{r}\,''(t) = (-9\cos(3t), -4\sin(2t), -2\sin(t))$ and $\vec{r}\,'''(t) = (27\sin(3t), 8\cos(2t), -2\cos(t))$.

WHAT IS MOTION?
The paradoxon of Zeno of Elea: "If we look at a body at a specific time, then the body is fixed. Having it fixed at each time, there is no motion". While one might wonder today a bit about Zeno's naivity, there were philosophers like Kant, Hume or Hegel, who thought seriously about Zeno's challenges. Also physisists continue to ponder about the question what is time and space. Today, the derivative or rate of change is defined as a **limit** $(\vec{r}(t+dt) - \vec{r}(t))/dt$ where dt approaches zero. If the limit exists it defines the velocity.



EXAMPLES OF VELOCITIES.

Electrons in Metals:	0.005 m/s
Person walking:	1.5 m/s
Car:	15-50 m/s
Signals in nerves:	40 m/s
Aeroplane:	70-900 m/s
Sound in air:	Mach1=340 m/s
Satellite:	1200 m/s
Speed of bullet:	1200-1500 m/s
Earth around the sun:	30'000 m/s
Sun around galaxy center:	200'000 m/s
Light in vacuum:	300'000'000 m/s

EXAMPLES OF ACCELERATIONS.

Train:	0.1-0.3 m/s^2
Car:	3-8 m/s^2
Space shuttle:	$\leq 3G = 30m/s^2$
Combat plane (F16) (blackout):	9G=90 m/s^2
Ejection from F16:	14G=140 m/s^2.
Free fall:	1G = 9.81 m/s^2
Electron in vaccum tube:	10^{15} m/s^2