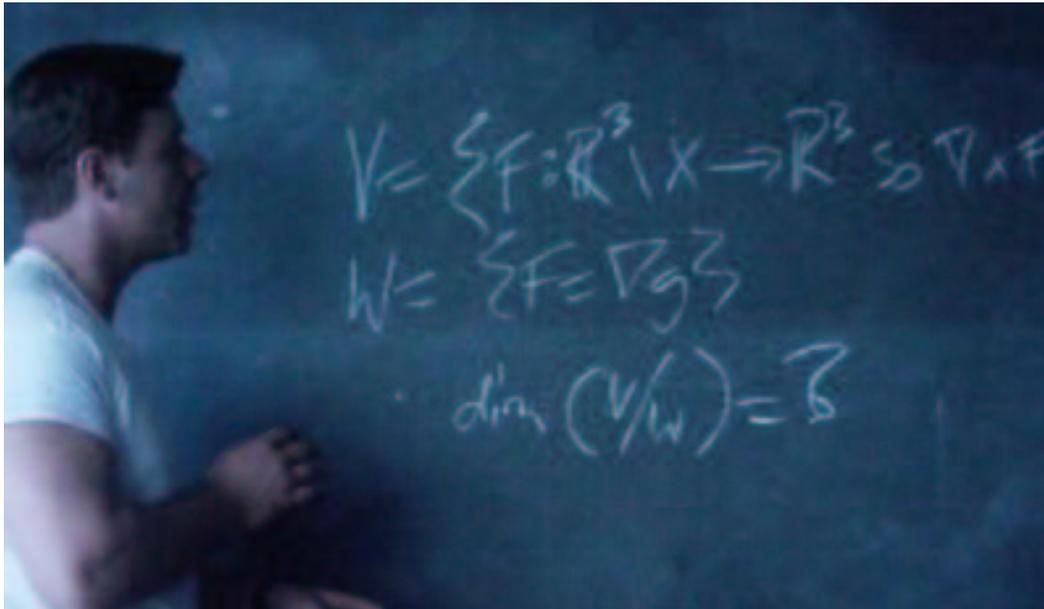


In this ICE, you solve Nash's problem, he gave to a multivariable calculus class. Remember Nash saying in the movie "A beautiful mind":

"It might take some of you a few months to solve it, for most of you however it might take a life time".

You will solve it here in 10 minutes ...



**NASH'S PROBLEM.** Find a subset  $X$  of  $\mathbf{R}^3$  with the property that if  $V$  is the set of vector fields  $F$  on  $\mathbf{R}^3 \setminus X$  which satisfy  $\text{curl}(F) = 0$  and  $W$  is the set of vector fields  $F$  which are conservative:  $F = \nabla f$ . Then, the space  $V/W$  should be 8 dimensional.

Remark. The meaning of the last sentence means that there should be 8 vectorfields  $F_i$  which are not gradient fields and which have vanishing curl outside  $X$ . Furthermore, you should not be able to write any of the 8 vectorfields as a sum of multiples of the other 7 vector fields.

You actually started already with a two dimensional version of the problem in a Homework:

**2D VERSION OF NASH'S PROBLEM.**

If  $X = \{0\}$  and  $V$  is the set of vector fields  $F$  on  $\mathbf{R}^2 \setminus X$  which satisfy  $\text{curl}(F) = 0$  and  $W$  is the set of vector fields  $F$  which are gradient fields, then  $\dim(V/W) = 1$ .

The vector field  $F$  is  $F(x, y) = (-y/(x^2 + y^2), x/(x^2 + y^2))$  is a gradient field in  $\mathbf{R}^2 \setminus X$  but not in  $\mathbf{R}^2$ .

Now: What set  $X$  would you have to take to get  $\dim(V/W) = 8$ ?

The SOLUTION OF THE 3D VERSION OF NASH'S PROBLEM can now be obtained directly from the solution of the 2D version. How?