7/24/2002, ICE: GABRIEL'S TRUMPET **S-Math 21a**

SURFACES OF REVOLUTION. Consider a positive function $f(x)$ on an interval $[a, b]$ on the x axes and and rotate the graph around the x-axes. We obtain a **surface of revolution** parameterized by $\boxed{(u, v) \mapsto X(u, v) = (v, f(v)\cos(u), f(v)\sin(u))}$ on $R = [0, 2\pi] \times [a, b]$.

SURFACE AREA. We have $X_u = (0, -f(v)\sin(u), f(v)\cos(u))$, $X_v = (1, f'(v)\cos(u), f'(v)\sin(u))$ and $X_u \times X_v = (-f(v)f'(v), f(v)\cos(u), f(v)\sin(u)) = f(v)(-f'(v), \cos(u), \sin(u))$ which has the length $|X_u \times X_v| = |f(v)|\sqrt{1 + f'(v)^2}$. The surface area of such a surface of revolution is therefore $\boxed{2\pi \int_a^b |f(v)|\sqrt{1 + f'(v)^2}\, dv.}$

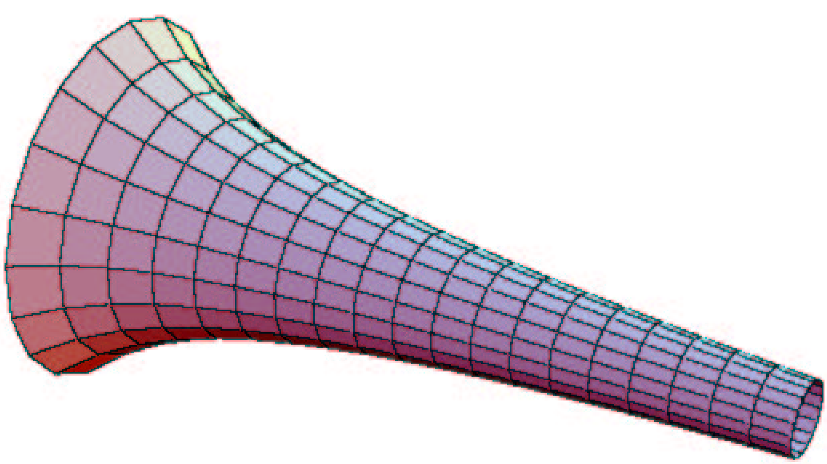
VOLUME. If we cut through the surface perpendicular to the x-axes, we obtain a disc of radius $f(x)$ and area $\pi f(x)^2$. The part of the surface between x and $x + dx$ has volume $f(x)^2 dx$. Therefore: The interior of a surface of revolution has volume $\boxed{\int_a^b \pi f(x)^2\, dx.}$

GABRIEL'S TRUMPET. The surface of revolution defined by $f(x) = 1/x$ on the interval $[1, \infty)$ is called **Gabriel's trumpet**.

Volume: The volume is

Surface area: Fill in the rest: The surface area is

$\geq 2\pi \int_1^\infty \frac{1}{x}\, dx = 2\pi \log(x)|_1^\infty = \infty.$



We conclude: the Gabriel trumpet is a surface of finite volume but with infinite surface area! You can fill the trumpet with a finite amount of paint, but this paint does not suffice to cover the surface of the trumpet!