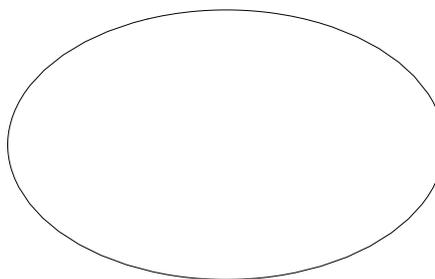


1) Look at the ellipse $C : r(t) = (a \cos(t), b \sin(t))$. By Green's theorem, the line integral of the vector field $F(x, y) = (0, x)$ along C gives the area A of the ellipse (which we know to be $A = \pi ab$).



$$r'(t) =$$

Let's do the calculation: $F(r(t)) \cdot r'(t) =$

$$\int_0^{2\pi} F(r(t)) \cdot r'(t) dt =$$

YOU SEE: The area calculation via **Green's theorem** was simpler than evaluating the 2D integral:

$$\int_{-a}^a \int_{-b\sqrt{1-x^2/a^2}}^{b\sqrt{1-x^2/a^2}} dy dx .$$

However, it would still have been possible to do the later.

2) Look at the deformed ellipse $C : r(t) = (a \cos(t) + 3 \sin(3t), b \sin(t))$. What is the area of this curve? Now, the 2D integral would even be **more difficult** to do.



$$r'(t) =$$

Do the calculation $F(r(t)) \cdot r'(t) =$

$$\int_0^{2\pi} F(r(t)) \cdot r'(t) dt =$$

3) Look at the curve $C : r(t) = (a \cos(t) + 5 \cos(5t), b \sin(t))$. What does Green's theorem say about the areas of regions defined by this curve? (Watch the orientation!)

