Solutions to Problem Set 2

July 10, 2002

Part 1

1. The function $f(x, y) = \log(x + y - 1)$ is defined when the argument $x + y - 1 > 0$ so the domain of f is $\{(x, y) | x + y > 1\}$. The range of f is all of $\mathbb{R}$.

2. This surface is a saddle.

5. The parametric form of the line is

$$
\begin{aligned}
x &= 1 + t \\
y &= 3 + 2t \\
z &= 2 + t
\end{aligned}
$$

(1)

Thus,

$$
\begin{aligned}
z^2 - x^2 = (2 + t)^2 - (1 + t)^2 &= (4 + 4t + t^2) - (1 + 2t + t^2) \\
&= 3 + 2t \\
&= y
\end{aligned}
$$

Part 2

1. a. $(r, \theta, z) = (\sqrt{2}, -\frac{\pi}{4}, 4)$ b. $(x, y, z) = (1, \sqrt{3}, 4)$

2. a. $(\rho, \theta, \phi) = (\sqrt{3}, \frac{\pi}{4}, \arctan(\sqrt{3}))$ b. $(x, y, z) = (1, \sqrt{3}, 0)$

3. To visualize this surface consider the trace in the yz-plane. Graphing this trace is a problem in 2-dimensional polar coordinates. The trace is a circle of radius $\frac{1}{2}$ the is tangent to z-axis at the origin. The surface is then achieved by rotating this circle about the z-axis. Thus, the surface looks a bit like a donut, but without a hole.

5. $x^2 + y^2 = r^2$ so $x^2 + y^2 - 5 = z^2$ implies that $r^2 - 5 = z^2$ which is the equation for the surface in cylindrical coordinates.