

This is part 3 (of 3) of the weekly homework. It is due August 13 in the mailbox of Jon. More problems to this lecture can be found on pages 983-984 and 987-988 in the book.

SUMMARY.

- $\operatorname{div}(F)(x, y, z) = F_x + F_y + F_z$ **divergence**. Is a scalar field.
- $\int \int \int_E \operatorname{div}(F) dV = \int \int_S F \cdot dS$ **divergence theorem**.

- 1) (4 points, compare problems 3-6 in 13.8) Verify that the divergence theorem is true for the vector field $F(x, y, z) = (3x, xy, 2xz)$ on the unit cube $[0, 1] \times [0, 1] \times [0, 1]$.
- 2) (4 points, compare problems 3-6 in 13.8) Verify that the divergence theorem is true for the vector field $F(x, y, z) = (xy, yz, zx)$ on the solid cylinder $x^2 + y^2 \leq 1, 0 \leq z \leq 1$.
- 3) (4 points, compare problems 7-15 in 13.8) Use the divergence theorem to calculate the flux of $F(x, y, z) = (x^3, y^3, z^3)$ through the sphere $x^2 + y^2 + z^2 = 1$.
- 4) (4 points, compare problem 19 in 13.8)
 - a) Verify that $\operatorname{div}(E) = 0$ away from the origin if E is the electric field $E(\vec{x}) = \vec{x}/|\vec{x}|^3$.
 - b) An electric charge at 0 generates the field E as in a). What is the flux of F through the unit sphere S ?
 - c) What is the flux of E through any other sphere containing the origin $(0, 0, 0)$ inside?
 - d) What is the flux of the electric field $F(\vec{x}) = \sum_{i=1}^n E(\vec{x} - \vec{x}_i)$ through S generated by n charges located at points \vec{x}_i inside S ?
- 5) (4 points) Find $\int \int \int_S F \cdot dS$, where $F(x, y, z) = (x, y, z)$ and S is the outwardly oriented surface obtained by removing the cube $[1, 2] \times [1, 2] \times [1, 2]$ from the cube $[0, 2] \times [0, 2] \times [0, 2]$.

CHALLENGE PROBLEM:

We show that Green's theorem in the plane is equivalent to Gauss theorem in the plane:



a) Let $G(x, y)$ be the vector field orthogonal to $F(x, y)$. Show that $\operatorname{div}(G) = \operatorname{curl}(F)$.

b) Show that the line integral $\int_C F dr$ along a curve C is the same as the flux integral $\int_C G \cdot dn$.

SUPER CHALLENGE PROBLEM:



Formulate the divergence theorem in arbitrary dimensions.