This is part 1 (of 3) of the weekly homework. It is due August 13 in the mailbox of Jon. More problems to this lecture can be found on pages 951-952 and 958-959 in the book.

SUMMARY.

- $\mathrm{curl}(P,Q) = Q_x - P_y$. **Curl** for 2D vector fields.
- C **positively oriented boundary** of the region D:
  (the region is "to the left" when you follow the boundary).
- $\boxed{\int_C F \cdot dr = \int\int_D \mathrm{curl}(F)\, dA}$ **Greens theorem**.
  Written out: $\int_C F(r(t)) \cdot r'(t)\, dt = \int\int_D (Q_x - P_y)\, dxdy$.
- $\int_C x\, dy$ **area** of D, C is the positively oriented boundary of D:
  Example: C unit circle: $x(t) = \cos(t), dy = \cos(t)dt$, Area $= \int_0^{2\pi} \cos^2(t)\, dt = \pi$.

1) (4 points, compare problems 1-4 in 13.4) Evaluate the line integral of the vector field $F(x,y) = (xy^2, x^2)$ along the rectangle with vertices $(0,0),(2,0),(2,3),(0,3)$ in two ways. Do this by calculating the line integral.

2) (4 points, compare problems 1-4 in 13.4) Recalculate the line integral in the previous problem using Green's theorem.

3) (4 points) Find the area of the region bounded by the hypocycloid $r(t) = (\cos(t)^3, \sin(t)^3)$ using Green's theorem.

4) (4 points) Verify that if C is the line segment connecting the point (x_1, y_1) to the point (x_2, y_2), then $\int_C x\, dy - y\, dx = x_1 y_2 - x_2 y_1$.

5) (4 points) Use 4) to verify that if $(x_1, y_1), ..., (x_n, y_n)$ are the vertices of a polygon in the plane, then $A = \frac{1}{2}[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + ... + (x_ny_1 - x_1y_n)]$ is the area of the polygon.

CHALLENGE PROBLEM:

(Compare problem 22 in 13.4) Let D be a region bounded by a simple closed path C in the plane. Use Green's theorem to prove that the coordinates of the center of mass are $(\int_C x^2 dy/(2A), -\int_C y^2 dx)/(2A)$, where A is the area of D.

SUPER CHALLENGE PROBLEM:

The planimeter calculates the area: the **planimeter vector field** $F(x,y) = (P(x,y), Q(x,y))$ is defined by attaching a unit vector orthogonal to the vector $(x-a, y-b)$ at (x,y), where (a,b) is the "knee" of the planimeter. The wheel rotation is the line integral of F along the boundary of R. By **Green's theorem**, this integral is the double integral of curl(F) over R. The planimeter vector field is explicitly given by $F(x,y) = (P(x,y), Q(x,y)) = (-(y - b(x,y)), (x - a(x,y)))$. Furthermore, $\mathrm{curl}(F) = Q_x - P_y$ is equal to $2 + (-a_x - b_y)$ which is 2 plus the curl of the vector field $G(x,y) = (b(x,y), -a(x,y))$. Show that $\mathrm{curl}(G) = -1$. For more information see http://www.math.duke.edu/education/ccp/materials/mvcalc/green/