This is part 3 (of 3) of the weekly homework. It is due August 6 at the beginning of class. More problems to this lecture can be found on pages 943-945 in the book.

SUMMARY.

- We say f **is conservative** or f **is a gradient field** if $F(x,y,z) = \nabla f(x,y,z)$.
- $\int_C \nabla f \; dr = f(r(b)) - f(r(a))$ **fundamental theorem of line integrals**: by chain rule: $\int_a^b \nabla f(r(t)) \cdot r'(t) \; dt = \int_a^b \frac{d}{dt} f(r(t)) \; dt$. Apply the fundamental theorem of calculus.
- If F is conservative in the plane then the line integrals do not depend on the path.
- If F is conservative in the plane and C is a closed curve, then $\int_C F \; dr = 0$.

1) (4 points) Calculate the line integral $\int_C 2x\sin(y)dx + (x^2\cos(y) - 3y^2) \; dy$ along a straight line from $(-1,0)$ to $(5,1)$.
Hint. You can do this easier using the fundamental theorem of line integrals (FTL).

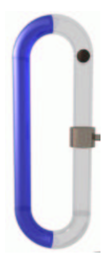
2) (4 points, compare problem 33 in 13.3) Let $F(x,y) = (-y/(x^2+y^2), x/(x^2+y^2))$. Let C : $r(t) = (\cos(t), \sin(t)), t \in [0, 2\pi]$.
a) What is $\int_C F \cdot dr$?
b) Let $f(x,y) = \arctan(y/x)$. Verify that $\nabla f = F$.
c) Why do a) and b) not contradict?

3) (4 points, compare problem 26 in 13.3) Let $F = \nabla f$ and $f(x,y) = \sin(x-2y)$. Find curves C_1 and C_2 that are not closed and satisfy the equation $\int_{C_1} F \cdot dr = 0$ and $\int_{C_2} F \cdot dr = 1$.

4) (4 points) Calculate the line integral of $F(x,y,z) = (-y, x, z^2)$ along the path $r(t) = (\cos(17t), \sin(17t), t)$, $t \in [0, 2\pi]$.
Hint. This can be done directly. It might be easier however to split up the field as a sum of two fields, where one is conservative.

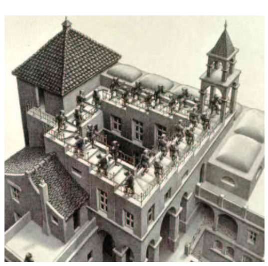
5) (4 points) Verify that the vector field $F(x,y,z) = (y, x, xyz)$ is not conservative.
Hint. If $F(x,y,z) = (P,Q,R)$ is conservative, then $P_y = Q_x, P_z = R_x, Q_z = R_y$.

CHALLENGE PROBLEM:

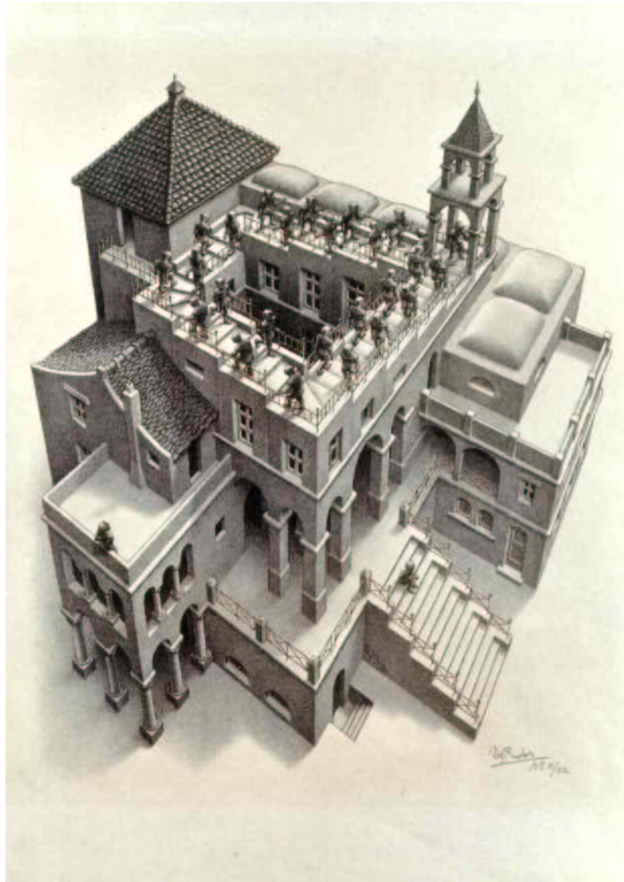


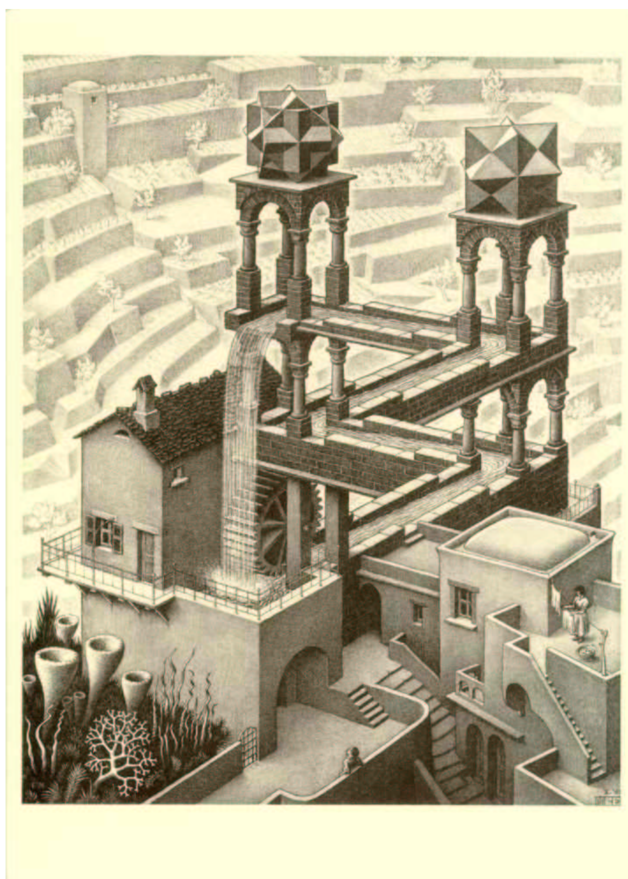
Consider a O shaped pipe which is filled only on the right side with water. A wooden ball falls on the right hand side in the air and moves up in the water. Why does this "perpetuum mobile" not work?

SUPER CHALLENGE PROBLEM:



What is wrong with the Escher picture to the left which describes a stair in which people always walk down? The figure suggests the existence of a force field which is not conservative.





Is the gravitational field really conservative? Escher's pictures suggest: not always ...