

This is part 2 (of 3) of the weekly homework. It is due August 6 at the beginning of class. More problems to this lecture can be found on pages 933-935 in the book.

SUMMARY.

- $\int_C F \cdot dr = \int_a^b F(r(t)) \cdot r'(t) dt$ **line integral** of F along curve $C : t \mapsto r(t)$.
- Example: $C : r(t) = (\cos(t), \sin(t)), t \in [0, 2\pi]$ (circle), $F(x, y) = (-y, x)$. $\int_C F \cdot dr = \int_F(r(t)) \cdot r'(t) dt = \int_0^{2\pi} (-\sin(t), \cos(t)) \cdot (-\sin(t), \cos(t)) dt = \int_0^{2\pi} 1 dt = 2\pi$.

- 1) (4 points, compare problems 1-14 in 13.2) Let C be the circle $x^2 + y^2 = 16$ and $F(x, y) = (x, y^4)$. Calculate the line integral $\int_C F \cdot dr$.
- 2) (4 points, compare problems 15-18 in 13.2) Let C be the space curve $r(t) = (\cos(t), \sin(t), t)$ for $t \in [0, 1]$ and let $F(x, y, z) = (y, x, 5)$. Calculate the line integral $\int_C F \cdot dr$.
- 3) (4 points) Calculate the line integral $F(x, y) = (x/\sqrt{x^2 + y^2}, y/\sqrt{x^2 + y^2})$ along the unit circle $r(t) = (\cos(t), \sin(t))$.
- 4) (4 points, problem 32 in 13.2) Find the work done by the force field $F(x, y) = (x \sin(y), y)$ on a particle that moves along the parabola $y = x^2$ from $(-1, 1)$ to $(2, 4)$.
- 5) (4 points, problem 38 in 13.2) A current I in a wire along the z axes produces a magnetic field B whose field lines rotate around the wire. Ampere's law states $\int_C B \cdot dr = \mu I$, where μ is a constant. Using circles with radius $r(t) = (\cos(t), \sin(t), 0)$ around the wire, show that the field strength $|B(r)|$ of the magnetic field vector $B(r)$ satisfies $|B(r)| = \mu I / (2\pi r)$.

CHALLENGE PROBLEM:



Let $F(x, y, z, w) = (x, y, z, w)$. Calculate the line integral along a straight line connecting $(0, 0, 0, 0)$ with $(1, 1, 1, 1)$.

SUPER CHALLENGE PROBLEM:



Does the line integral concept still make sense with a different dot product $v \cdot w = g_1 v_1 w_1 + g_2 v_2 w_2 + g_3 v_3 w_3$?