

This is part 1 (of 3) of the weekly homework. It is due August 6 at the beginning of class. More problems to this lecture can be found on pages 922-924 in the book.

## SUMMARY.

- $F(x, y) = (P(x, y, z), Q(x, y, z))$  **vector field in the plane.**
- $F(x, y, z) = (P(x, y, z), Q(x, y, z), R(x, y, z))$  **vector field in space.**
- $F(x, y, z) = \nabla f(x, y, z)$  **gradient field.**
- $F(x, y)$  vector field. **Flow line:** curve  $r(t)$  such that  $r'(t)$  is parallel to  $F(r(t))$ . Example:  $r(t) = (a \cos(t), a \sin(t))$  are flow lines to  $F(x, y) = (-y, x)$ .

- 1) (4 points, compare problems 1-10 in 13.1) Sketch the vector field  $F(x, y) = (y/(x^2+y^2), -x/(x^2+y^2))$ .
- 2) (4 points, compare problems 21-24 in 13.1) Determine the gradient field of  $f(x, y) = \sin(x+y)$ . Draw a contour map of  $f$  and sketch also the vector field  $F(x, y) = \nabla f$ .
- 3) (4 points) Can  $(P(x, y), Q(x, y)) = (xy, x^2)$  be a gradient field? Hint: Assume it is  $P = f_x, Q = f_y$ , then calculate  $f_{xy}$  in two ways.
- 4) (4 points) For which values of  $(a, b)$  is the vector field  $F(x, y) = (ax^3y + by^2, x^4 + yx)$  a gradient vector field?
- 5) (4 points, compare problem 34 in 13.1) Given the vector field  $F(x, y) = (1, x)$ . Sketch some flow lines. What shape do these flow lines appear to have?

## CHALLENGE PROBLEM:



An electric charge has a field  $F = \nabla f$  where  $f(x, y, z) = 1/\sqrt{|x^2 + y^2 + z^2|}$ . Assume you have three positive electric charges located on an equilateral triangle with side length 1. Without writing down any formula, sketch the field  $F$  in the plane spanned by the charges. The field is the sum of the fields of the three charges.

## SUPER CHALLENGE PROBLEM:



Get a current wind velocity chart from the US. Use the web, type "GOES 8 wind" or "US wind map" into google. Can you locate places where the wind velocity  $F(x, y)$  is zero? Is the wind velocity field a gradient field?