7/24/2002 3D INTEGRALS, SURFACE AREA (Section 12.6-12.9) S-Math 21a

This is part 2 (of 2) of the weekly homework. It is due July 30 at the beginning of class. More problems to this lecture can be found on pages 881-882, 890-892 and 898-900 in the book.

SUMMARY.

- $r(u,v)$ **parametric surface** defined on $[a,b]\times[c,d]$ has **area** $\int_a^b \int_c^d |r_u(u,v)\times r_v(u,v)|\ dudv$.
- $\int\int\int_E f dV = \int_o^p \int_q^r \int_s^t f(x,y,z)\ dzdydx$ **triple integral**.
- $V(E) = \int\int\int_E 1 dV$ **volume** of body E.
- $\frac{1}{V(E)}\int\int\int_E f\ dV$ **average value** of f over E.
- $\int\int\int_E f\ dV = \int_\gamma^\delta \int_\alpha^\beta \int_a^b f(\rho\sin(\phi)\cos(\theta),\rho\sin(\phi)\sin(\theta),\rho\cos(\phi))\rho^2\sin(\phi)\ d\rho d\theta d\phi$ integral over **spherical wedge**

1) (6 points, compare problem 10 in 12.6) Find the area of the paraboloid $x = y^2 + z^2$ that lies inside the cylinder $y^2 + z^2 = 9$.

2) (6 points, compare problem 26 in 12.6) A torus is parametrized by $r(t) = (2\cos(u) + \cos(u)\cos(v), 2\sin(u) + \sin(u)\cos(v), \sin(v))$. Find the area of the torus.

3) (6 points, compare problem 6 in 12.7) Evaluate the following iterated integral $\int_0^1 \int_0^z \int_0^y ze^{-y^2}\ dxdydz$.

4) (6 points, compare problem 18 in 12.7) Find the volume of the solid bounded by the paraboloids $z = x^2 + y^2$ and $z = 18 - x^2 - y^2$.

5) (6 points, compare 33-36 in 12.7) Find the mass and center of mass of a solid hemi sphere of radius 1 if the density at any point is proportional to its distance from its base. (In other words, calculate $\int\int\int_E z\ dxdyz$, where E is the part of the sphere with $z \geq 0$.)

CHALLENGE PROBLEM:

Find the volume of the intersection of two cylinders $y^2 + z^2 = 1$ and $x^2 + z^2 = 1$. You have a picture on page 882 in the text.

Hint. Look what happens, when you cut the body at a fixed z value and calculate the area of this section.

SUPER CHALLENGE PROBLEM:

Use a quadruple integral to find the volume of the hyper-sphere $x^2 + y^2 + z^2 + w^2 = r^2$ in $\mathbf{R}^4$.
Hint. If you slice up the hyper-sphere at w, you get a sphere of radius $\sqrt{r^2 - w^2}$. Integrate the volume of the sphere from $w = -r$ to $w = r$ using substitution.

Can you find a formula for the volume of the n dimensional hyper-sphere?