This is part 1 (of 2) of the weekly homework. It is due July 30 at the beginning of class. More problems to this lecture can be found on pages 847-848, 853-854, 861-862 and 867-868 in the book.

SUMMARY. $dA = dxdy$ area element.

- $\int\int_R f\ dA = \int_a^b \int_c^d f(x,y)\ dydx$ **double integral** over rectangle.
- $\int\int_R f\ dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\ dydx$ double integral over **type I region**.
- $\int\int_R f\ dA = \int_a^b \int_{h_1(y)}^{h_2(y)} f(x,y)\ dxdy$ double integral over **type II region**.
- $A(R) = \int\int_R 1\ dA$ **area** of R.
- $\frac{1}{A(R)} \int\int_R f\ dA$ **average value** or **mean** of f on R.
- $\int\int_R f(x,y)dxdy = \int_\alpha^\beta \int_a^b f(r\cos(\theta), r\sin(\theta))\ rdrd\theta$ **integral in polar coordinates**.

1) (6 points, compare problems 3-10 in 12.2) Calculate the iterated integral $\int_1^4 \int_0^2 (x + \sqrt{y})\ dxdy$.

2) (6 points, compare problem 20 in 12.2) Find the volume of the solid lying under the paraboloid $z = x^2 + y^2$ and above the rectangle $R = [-2,2] \times [-3,3]$.

3) (6 points, compare problems 1-6 in 12.3) Calculate the iterated integral $\int_0^1 \int_x^{2-x} (x^2 - y)\ dydx$. Sketch the correspdinging type I region. Write this integral as integral over a type II region and compute the integral again.

4) (6 points, compare problem 10 in 12.4) Evaluate the following iterated integral by converting it into polar coordinates

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \sqrt{x^2 + y^2}\ dydx\ .$$

5) (6 points, compare problems 23-24 in 12.4) Find the average value of $f(x,y) = x^2 + y^2$ on the annulus $1 \leq |(x,y)| \leq 2$.

CHALLENGE PROBLEM:

The integral $\int_0^1 \arccos(\sqrt{x})\ dx$ can be written as a double integral $\int_0^1 \int_0^{\arccos(\sqrt{x})} dydx$. Calculate this integral.

SUPER CHALLENGE PROBLEM:

Calculate $\int\int_{\mathbf{R}^2} e^{-x^2-y^2}\ dxdy$ and use this to calculate the integral $\int_{-\infty}^\infty e^{-x^2}\ dx$.

Hint. The function $f(x) = e^{-x^2}$ is known to have no anti-derivative which can be expressed with "known functions" like $\exp, \log, \sin$ etc. You can nevertheless find a closed solution for the definite integral $\int_{-\infty}^\infty e^{-x^2}\ dx$.