7/17/2002 CHAIN RULE, D_u, TANGENT PLANES (11.5/11.6) S-Math 21a

This is part 2 (of 3) of the weekly homework. It is due July 23 at the beginning of class. More problems to this lecture can be found on pages 796-798 and 808-810 in the book.

SUMMARY.

- $\nabla f(x,y,z) = (f_x(x,y,z), f_y(x,y,z), f_z(x,y,z))$ **gradient**.
- $f(x,y,z)$ function of three variables, $r(t)$ **curve**, $\frac{d}{dt}f(r(t)) = \nabla f(r(t)) \cdot r'(t)$ **chain rule**.
- u unit vector, $D_u f(x,y,z) = \nabla f(x,y,z) \cdot u$ **directional derivative**.
- $n = \nabla f(x_0,y_0,z_0)$ is orthogonal to $f(x,y,z) = c$, where $c = f(x_0,y_0,z_0)$. **"Gradients are orthogonal to level curves rsp. level surfaces."**
- $n \cdot (x,y,z) = d = n \cdot (x_0,y_0,z_0)$ **tangent plane** to $f(x,y,z) = c$ at point (x_0,y_0,z_0).
- $\nabla f(\vec{x}_0) \cdot (\vec{x} - \vec{x}_0) = 0$ **tangent plane** to $f(\vec{x}) = c = f(\vec{x}_0)$ (vector notation).

1) (4 points, compare problems 1-14 in 11.5) Let $r(t) = (\cos(t), \sin(t), t)$ and $T(x,y,z) = x^2 + y^2 + 2z^2$. Find $\frac{d}{dt}T(r(t))$ at $t = 0$ in two different ways. First by differentiating the function $g(t) = T(r(t))$ and second, using the chain rule.

2) (4 points, compare problems 46 in 11.6) Find the equation for the tangent plane at the surface $x^2 + 2y^2 - z^2 = 1$ at the point $(2,1,0)$.

3) (4 points, compare problem 24 in 11.6) Find the directions in which the directional derivative of $f(x,y) = x^2 + \sin(xy)$ at the point $(1,0)$ has the value 1.

4) (4 points, compare problem 51 in 11.6) Verify that the surface $z^2 = x^2 + 3y^2$ and $x^2 + y^2 + z^2 = 1$ are orthogonal at every point of intersection.

5) (4 points, compare problem 30 in 11.6) Suppose that your are climbing a hill whose shape is given by the equation $z = 1000 - 0.01x^2 - 0.02y^2$ and you are standing at a point with coordinates $(60, 100, 764)$.
a) In which direction should you proceed initially in order to reach the top of the hill fastest.
b) If you climb in that direction, at what angle above the horizontal will you be climbing initially?

CHALLENGE PROBLEM:

The partial derivatives of the function $f(x,y) = (xy)^{1/3}$ exists at every point but the directional derivatives in all other directions don't exist. What is going on?

SUPER CHALLENGE PROBLEM:

Extend the notion of "tangent plane" to 5-dimensional hyper-surfaces $f(x,y,z,u,v,w) = c$ in 6-dimensional space. Find two different quadrics of the form $ax^2 + by^2 + cz^2 + dv^2 + ew^2 = k$, where a,b,c,d,e,k are constants, such that they are orthogonal to each other at each point of intersection.