7/16/2002 PARTIAL DERIVATIVES (Section 11.1-11.4) **S-Math 21a**

This is part 1 (of 3) of the weekly homework. It is due July 23 at the beginning of class. More problems to this lecture can be found on pages 756-759 and 788-789 in the book.

SUMMARY.

- $f(x,y) = c$ **level curve**, a set of level curves forms a **contour map**.
- $\frac{\partial f}{\partial x} f(x,y) = f_x(x,y)$ **partial derivative**.
- $\nabla f(x,y) = (f_x(x,y), f_y(x,y))$ **gradient**.
- **Clairot**: $f_{xy} = f_{yx}$ for smooth functions.
- $L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$ **linear approximation** of f at (x_0,y_0).
  Can approximate or estimate $f(x,y)$ by $L(x,y)$ near $f(x_0,y_0)$.
- $L(\vec{x}) = f(\vec{x}_0) + \nabla f(\vec{x}_0) \cdot (\vec{x} - \vec{x}_0)$ **linear approximation** in vector notation.

1) (4 points, compare problems 15-22 in 11.1) Sketch a contour map of the function $f(x,y) = x^2 + 9y^2$. Find the **gradient vector** $\nabla f = (f_x, f_y)$ of f at the point $(1,1)$ and draw it.

2) (4 points, compare problem 70 in 11.3) The **ideal gas law** for a gas with fixed mass m at temperature T, pressure P and volume V is $PV = mRT$, where R is the gas constant. Show that $\frac{\partial P}{\partial V}\frac{\partial V}{\partial T}\frac{\partial T}{\partial P} = -1$. Note: the relation $PV = mRT$ leads to functions $P(V,T) = mRT/V$. Analogously there are functions $T(V,P), V(P,T)$. $\frac{\partial P}{\partial V}$ is a short hand notation for $\frac{\partial P(V,T)}{\partial V}$.

3) (4 points, compare problems 64-68 in 11.3) Verify that $f(x,t) = e^{-rt} f(x+ct)$ satisfies the equation $f_t(x,t) = cf_x(x,t) - rf(x,t)$. This equation is an example of a partial differential equation (PDE). It is called the **advection equation**.

4) (4 points, compare problem 66 in 11.4) **Cobb and Douglas** found in 1928 empirically a formula $P(L,K) = bL^\alpha K^\beta$ giving the total production P of an economic system as a function of the amount of labor L and the capital investment K. By fitting data, they got $b = 1.01, \alpha = 0.75, \beta = 0.25$. Verify that the function $P(L,K)$ satisfies the PDE $LP_L + KP_K = P$.

5) (4 points, compare problem 13 in 11.4) Find the linear approximation $L(x,y)$ of the function $f(x,y) = \sqrt{10 - x^2 - 5y^2}$ at $(2,1)$ and use it to estimate $f(1.95, 1.04)$.

CHALLENGE PROBLEM:

Verify that the **N-wave** function $f(t,x) = \frac{x}{t}\frac{\frac{1}{t}e^{-x^2/(4t)}}{1+\frac{1}{t}e^{-x^2/(4t)}}$ satisfies the PDE $f_t + ff_x = f_{xx}$ called **Burger's equation**.

SUPER CHALLENGE PROBLEM:

Choice: either
a) Given $g(x,y) = x^2$ and $h(x,y) = y^3 + x^2$. Is there a function $f(x,y)$ such that $\nabla f(x,y) = (f_x(x,y), f_y(x,y)) = (g(x,y), h(x,y))$?
b) Verify that the **soliton** $f(t,x) = \frac{a^2}{2}\cosh^{-2}(\frac{a}{2}(x - a^2 t))$ is a solution of the **KDV equation** $f_t + 6ff_x + f_{xxx} = 0$.