7/09/2002 CURVES, VELOCITY (Section 10.1/10.2) **S-Math 21a**

This is part 1 (of 2) of the homework for the third week. It is due July 16 at the beginning of class. More problems to this lecture can be found on pages 710-711 and 716-717 in the book.

SUMMARY.

- $r(t) = (x(t), y(t), z(t))$, $t \in [a,b]$ **curve** in space, $[a,b]$ domain.
- $t \mapsto r(t)$ is a vector function called **parametric representation** of the curve.
- $r'(t) = (x'(t), y'(t), z'(t))$ **velocity**, $T(t) = r'(t)/|r'(t)|$ **unit tangent vector**.
- $r''(t) = (x''(t), y''(t), z''(t))$ **acceleration**.
- $R(t) = \int_a^t r(s)\ ds = (\int_a^t x(s)\ ds, \int_a^t y(s)\ ds, \int_a^t z(s)\ ds)$ **antiderivative**, satisfies $R'(t) = r(t)$.

1) a) (3 points, compare problem 12 in 10.1) Draw the plane curve $r(t) = (x(t), y(t)) = (t^3, t^2)$ for $t \in [-1, 1]$ and calculate its velocity $r'(t)$ as well as its acceleration.
b) (3 points) Sketch the space curve $r(t) = (\sin(t), t^3, \cos(t))$ and calculate its velocity $r'(t)$ as well as the unit tangent vector $T(t)$ to the curve at time t.

2) (6 points, compare problems 35-26 in 10.2) A device in a car can measure the acceleration $r''(t) = (\cos(t), -\cos(3t), t0.2)$ at time t. Assume that the car is at the origin at time $t = 0$ and has no speed at $t = 0$, what is its position $r(t)$ at time t?

3) (6 points, compare problem 27 in 10.1) Show that the curve $r(t) = (x(t), y(t), z(t)) = (t^2, 1 + t, 1 + t^3)$ passes through the point $(1, 0, 0)$ but not through the point $(1, 2, -2)$. What is the velocity $r'(t)$ and the acceleration $r''(t)$ at $(1, 0, 0)$?

4) (6 points, compare problem 19 in 10.1) Verify that the curve $r(t) = (t\cos(t), t\sin(t), t^2)$ lies on the paraboloid $z = x^2 + y^2$. Use this fact to sketch the curve.

5) (6 points, compare problem 28 in 10.1) Find a vector function $r(t) = (x(t), y(t), z(t))$ which represents the curve obtained by intersecting the cylinder $x^2 + y^2 = 4$ with the surface $z = xy$. Write down the velocity $r'(t)$.
Hint: Find first $x(t), y(t)$ so that the first equation is satisfied, then get $z(t) = x(t)y(t)$.

CHALLENGE PROBLEM:

A closed curve in space is called a **knot**. Consider the space curve $r(t) = (\sin(3t), \cos(4t), \cos(5t))$. Find the smallest interval $[a, b]$ such that this curve is a knot. Sketch the curve.

Hint. Try first without technology. If needed, peek at the website http : //www.math.harvard.edu/jdg or type
ParametricPlot3D[{Sin[3t], Sin[4t], Sin[5t]}, {t, 0, 2Pi}]
in Mathematica.

SUPER CHALLENGE PROBLEM:

How could one verify that it is not possible to deform the knot $r(t) = (\sin(3t), \cos(4t), \cos(5t))$ into the trivial knot $r(t) = (\cos(t), \sin(t), 0)$ in such a way that during the deformation, the curve can never selfintersect?

Hint. Look at the possible types of closed curves which don't intersect the knot. How many different types are there for the trivial knot or for the given knot?