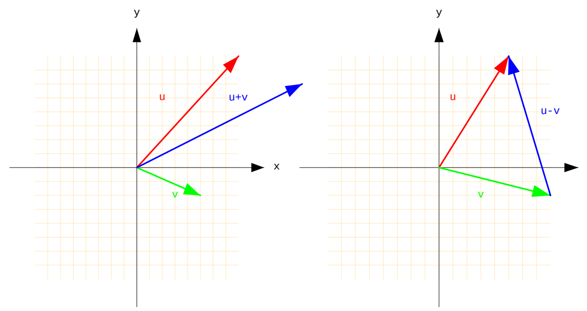
6/26/2002 VECTORS AND DOT PRODUCT (Section 9.2-9.3) S-Math 21a

This is part 2 (of 3) of the homework. It is due July 2 at the beginning of class. More problems to this lecture can be found on pages 659-661 and 666-667 in the book.

SUMMARY.

- **Vectors** $v = (v_1, v_2, v_3) = PQ$. **Points** are special vectors $P = OP$, with $O = (0, 0, 0)$.
- $v \cdot w = v_1w_1 + v_2w_2 + v_3w_3 = |v||w|\cos(\phi)$ **dot product**
- $|v \cdot w| = |v||w|\cos(\phi)$ **angle** ϕ between vectors.
- $\text{proj}_a(b) = a(a \cdot b)/|a|^2$ **projection** of b onto a.
- $\text{comp}_a(b) = |\text{proj}_a(b)| = |(a \cdot b)|/|a|$ **scalar projection** of b onto a.



1) (4 points, Problem 12 in 9.2) Let $a = (-1, 2)$ and $b = (5, 3)$ be two vectors in the plane attached at the origin $(0, 0)$. Draw them. Determine the sum $a + b$ as well as the difference $a - b$ and draw these two vectors also.

2) (4 points, Problem 20 in 9.2) Find a vector with the same direction as $v = (-2, 4, 2)$ but which has length 6.

3) (4 points, Problem 18 in 9.3) For which values of numbers b are the vectors $v = (-6, b, 2)$ and $w = (b, b^2, b)$ orthogonal?

4) (4 points, Problem 36 in 9.3) Find the angle between the diagonal of a cube and a diagonal of one of its faces.

5) (4 points, Compare Problem 21-14 in 9.3) Find the scalar and vector projection of the vector $b = (-4, 2, 2)$ onto the vector $a = (3, 0, 4)$.

CHALLENGE PROBLEM:

Verify that for any two vectors a and b, the inequality $|a - b| \geq ||a| - |b||$ holds.

Hint: Check both $|a - b| \geq |a| - |b|$ and $|a - b| \geq |b| - |a|$ using an inequality you know.

SUPER CHALLENGE PROBLEM:

Given three numbers g_1, g_2, g_3. Define a new **dot product** $(v, w) = g_1v_1w_1 + g_2v_2w_2 + g_3v_3w_3$. For $g_1 = g_2 = g_3 = 1$, this is the usual dot product.

Which properties of the usual dot product still hold for this generalization? For which g_1, g_2, g_3 could the dot product still serve to measure a reasonable "length" $|v| = \sqrt{(v, v)}$?