

This is part 1 (of 3) of the weekly homework. It is due July 2 at the beginning of class. The syllabus contains information on how to make use of the challenge problem. The super challenge problem is for fun only. If you have time, we encourage that you do more exercises on page 651-652.

SUMMARY.

- $d((x, y, z), (u, v, w)) = \sqrt{(x - u)^2 + (y - v)^2 + (z - w)^2}$ **distance**.
- $(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$ equation of a **sphere** with center (a, b, c) .
- **completion of the square:**
 $x^2 + ax = b \Leftrightarrow x^2 + ax + a^2/4 = b + a^2/4 \Leftrightarrow (x + a/2)^2 = b + a^2/4$.

- 1) (4 points, Compare Problems 5-6 in 9.1) Describe and sketch the surface in \mathbf{R}^3 represented by the equation $x + 2y = 4$.
- 2) (4 points, Problem 8 in 9.1) Find the distance from the point $P = (3, 7, -5)$
 - a) to the z axes, b) to the xy coordinate plane, c) to the origin.
- 3) (4 points, Problem 10 in 9.1) Find the equation of the sphere with center $(6, 5, -2)$ and radius $\sqrt{7}$. Describe the traces of this surface, its intersection with each of the coordinate planes.
- 4) (4 points, Problem 14 in 9.1) Find the center and radius of the sphere $4x^2 + 4y^2 + 4z^2 - 8x + 16y = 1$.
- 5) (4 points, Compare Problems 29-32 in 9.1) Write inequalities which describe the region of all points between (but not on) the spheres of radius 2 and 3 centered at the origin.

CHALLENGE PROBLEM:



(Descartes)

Show that the set of points in the plane for which the sum of the distances from two points $(-1, 0)$ and $(1, 0)$ is constant=3 forms an **ellipse**: $x^2/a^2 + y^2/b^2 = 1$.

Hint: Start with $\sqrt{(x - 1)^2 + y^2} + \sqrt{(x + 1)^2 + y^2} = 3$ and bring it into the form of an ellipse.

SUPER CHALLENGE PROBLEM:



(Newton)

An other distance in the plane is defined by $d((x, y), (u, v)) = |x - u| + |y - v|$. It is called the **taxi metric** because a taxi driver in a town, where all streets are parallel either to the x or y axes experiences this distance between two points. How does an ellipse look like in this metric? You can assume that the ellipse is defined as the set of points (x, y) which have the property that the sum of the distances to $(-1, 0)$ and $(0, 1)$ is 4.

Hint: You can explore this problem by taking a paper with a grid. Choose two points on the x axes a few grid-points apart and look at all grid-points which satisfy the requirement.