

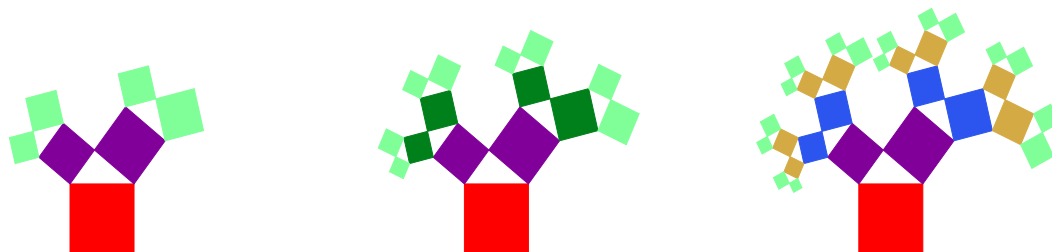
Lecture 1: A tree

In this first lecture, we look at a mathematical object and try to understand it. The object has been found by a Dutch math teacher in 1942. It will touch on a couple of mathematical topics. Let's start with a famous picture:

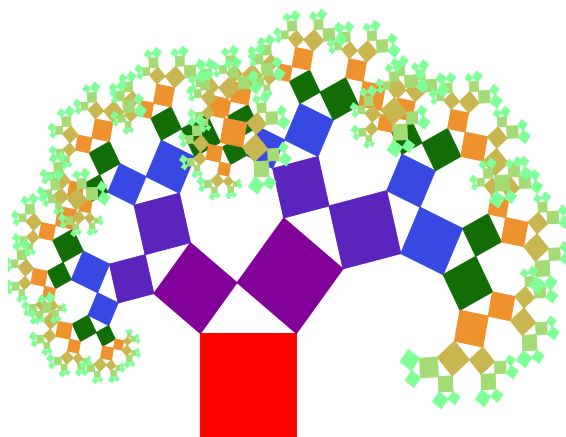


Basic question: Assume the base square has area 1, what is the relation between the areas of the three squares?

Now, we look at the above figure as a **rule for a construction**: the base square is expanded and becomes a list of three squares. This is best explained in a picture: we see stages 1, 2 and 3:



And here is the situation after 7 steps:



The limiting figure is called the **Pythagoras tree**:

Questions

- 1 What is the area of the tree in stage n ? Form a conjecture.
- 2 Can you prove the conjecture?
- 3 Here is a more challenging question: Do you think the limiting tree fits in a finite region? Why or why not?
- 4 What happens in extreme cases, like very symmetric or very acute angle cases?
- 5 Can you find an explicit bound if the triangle is an isosceles right triangle?
- 6 There is a relation with a logarithmic spiral. Can you find this?
- 7 Can you research about the literature? Especially find the publication of Albert E. Bosman?
- 8 Can you generalize the tree construction? What happens especially if the angle gets changed.
- 9 Are there higher dimensional versions?