

Lecture 1: Mathematical roots

Similarly, as one has distinguished the **canons of rhetorics**: memory, invention, delivery, style, and arrangement, or combined the **trivium**: grammar, logic and rhetorics, with the **quadrivium**: arithmetic, geometry, music, and astronomy, to obtain the seven **liberal arts and sciences**, one has tried to **organize all mathematical activities**.

Historically, one has distinguished **eight ancient roots of mathematics**. Each of these 8 activities in turn suggest a key area in mathematics:

counting and sorting	arithmetic
spacing and distancing	geometry
positioning and locating	topology
surveying and angulating	trigonometry
balancing and weighing	statics
moving and hitting	dynamics
guessing and judging	probability
collecting and ordering	algorithms

To morph these 8 roots to the 12 mathematical areas covered in this class, we complemented the ancient roots with calculus, numerics and computer science, merge trigonometry with geometry, separate arithmetic into number theory, algebra and arithmetic and turn statics into analysis.

Lets call this modern adaptation the

12 modern roots of Mathematics:

counting and sorting	arithmetic
spacing and distancing	geometry
positioning and locating	topology
dividing and comparing	number theory
balancing and weighing	analysis
moving and hitting	dynamics
guessing and judging	probability
collecting and ordering	algorithms
slicing and stacking	calculus
operating and memorizing	computer science
optimizing and planning	numerics
manipulating and solving	algebra

While relating **mathematical areas** with **human activities** is useful, it makes sense to select specific topics in each of this area. These 12 topics will be the 12 lectures of this course.

Arithmetic	numbers and number systems
Geometry	invariance, symmetries, measurement, maps
Number theory	Diophantine equations, factorizations
Algebra	algebraic and discrete structures
Calculus	limits, derivatives, integrals
Set Theory	set theory, foundations and formalisms
Probability	combinatorics, measure theory and statistics
Topology	polyhedra, topological spaces, manifolds
Analysis	extrema, estimates, variation, measure
Numerics	numerical schemes, codes, cryptology
Dynamics	differential equations, maps
Algorithms	computer science, artificial intelligence

Like any classification, this chosen division is rather arbitrary and a matter of personal preferences. The **2010 AMS classification** distinguishes 64 areas of mathematics. Many of the just defined main areas are broken off into even finer pieces. Additionally, there are fields which relate with other areas of science, like economics, biology or physics:

00 General
 01 History and biography
 03 Mathematical logic and foundations
 05 Combinatorics
 06 Lattices, ordered algebraic structures
 08 General algebraic systems
 11 Number theory
 12 Field theory and polynomials
 13 Commutative rings and algebras
 14 Algebraic geometry
 15 Linear/multi-linear algebra; matrix theory
 16 Associative rings and algebras
 17 Non-associative rings and algebras
 18 Category theory, homological algebra
 19 K-theory
 20 Group theory and generalizations

22 Topological groups, Lie groups
 26 Real functions
 28 Measure and integration
 30 Functions of a complex variable
 31 Potential theory
 32 Several complex variables, analytic spaces
 33 Special functions
 34 Ordinary differential equations
 35 Partial differential equations
 37 Dynamical systems and ergodic theory
 39 Difference and functional equations
 40 Sequences, series, summability
 41 Approximations and expansions
 42 Fourier analysis
 43 Abstract harmonic analysis
 44 Integral transforms, operational calculus

45 Integral equations
 46 Functional analysis
 47 Operator theory
 49 Calculus of variations, optimization
 51 Geometry
 52 Convex and discrete geometry
 53 Differential geometry
 54 General topology
 55 Algebraic topology
 57 Manifolds and cell complexes
 58 Global analysis, analysis on manifolds
 60 Probability theory and stochastic processes
 62 Statistics
 65 Numerical analysis
 68 Computer science
 70 Mechanics of particles and systems

74 Mechanics of deformable solids
 76 Fluid mechanics
 78 Optics, electromagnetic theory
 80 Classical thermodynamics, heat transfer
 81 Quantum theory
 82 Statistical mechanics, structure of matter
 83 Relativity and gravitational theory
 85 Astronomy and astrophysics
 86 Geophysics
 90 Operations research, math. programming
 91 Game theory, Economics Social and Behavioral Sciences
 92 Biology and other natural sciences
 93 Systems theory and control
 94 Information and communication, circuits
 97 Mathematics education

What are

fancy developments

in mathematics today? Michael Atiyah identified in the year 2000 the following **six hot spots**:

local	and	global
low	and	high dimension
commutative	and	non-commutative
linear	and	nonlinear
geometry	and	algebra
physics	and	mathematics

Also this choice is of course highly personal. One can easily add 12 other **polarizing** quantities which help to distinguish or parametrize different parts of mathematical areas, especially the ambivalent pairs which produce a captivating gradient:

regularity	and	randomness	discrete	and	continuous
integrable	and	non-integrable	existence	and	construction
invariants	and	perturbations	finite dim	and	infinite dimensional
experimental	and	deductive	topological	and	differential geometric
polynomial	and	exponential	practical	and	theoretical
applied	and	abstract	axiomatic	and	case based

An other possibility to refine the fields of mathematics is to **combine** different of the 12 areas. Examples are **probabilistic number theory**, **algebraic geometry**, **numerical analysis**, **geometric number theory**, **numerical algebra**, **algebraic topology**, **geometric probability**, **algebraic number theory**, **dynamical probability = stochastic processes**. Almost every pair is an actual field. Finally, lets give a short answer to the question: What is Mathematics?

Mathematics is the science of structure.

The goal is to illustrate some of these structures from a historical point of view.