Lecture 1: A tree

In this first lecture, we look a mathematical object and try to understand it. The object has been found by a Dutch math teacher in 1942. It will touch on a couple of mathematical topics. Lets start with a famous picture:
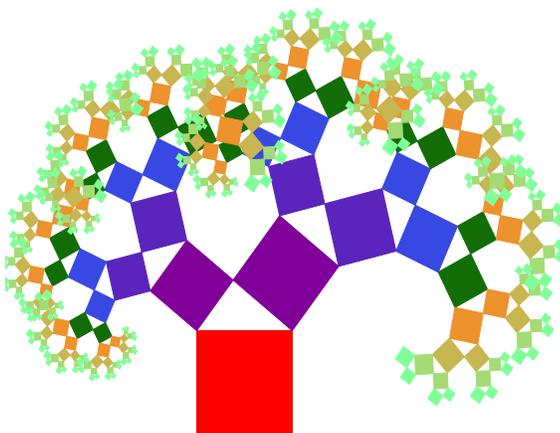

> Basic question: Assume the base square has area 1, what is the relation between the areas of the three squares?

Now, we look at the above figure as a **rule for a construction**: the base square is expanded and becomes a list of three squares. This is best explained in a picture: we see stages 1,2 and 3:
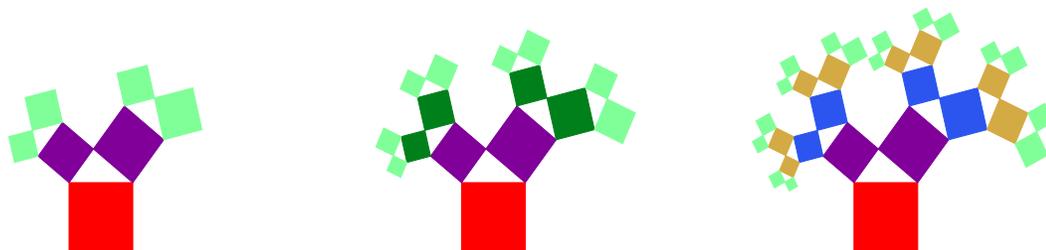
And here is the situation after 7 steps:

The limiting figure is called the **Pythagoras tree**:

Questions

1. What is the area of the tree in stage n? Form a conjecture.

2. Can you prove the conjecture?

3. Here is a more challenging question: Do you think the limiting tree fits in a finite region? Why or why not?

4. What happens in extreme cases, like very symmetric or very acute angle cases?

5. Can you find an explicit bound if the triangle is an isosceles right triangle?

6. There is a relation with a logarithmic spiral. Can you find this?

7. Can you research about the literature? Especially find the publication of Albert E. Bosman?

8. Can you generalize the tree construction? What happens especially if the angle gets changed.

9. Are there higher dimensional versions?