

## Lecture 10: Fractals

### 1. Objective

We want to compute the dimension of various objects in the plane.

### 2. Definition of Dimension

If we need  $n$  squares of side length  $r$  to cover an object  $X$ . the dimension is defined as

$$d = \frac{-\log(n)}{\log(r)} \quad \text{when } r \text{ gets zero.}$$

### 3. Dimension of a curve

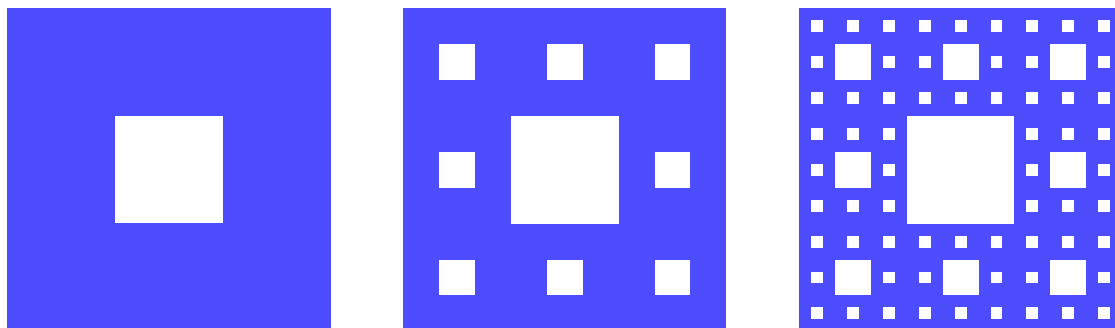
1) Assume a curve is given as the boundary of the unit square. How many squares of length  $r = 1/10$  do we need to cover the curve? If we call  $n$  this number, what is  $-\log(n)/\log(r)$ ?

### 4. Dimension of a region

2) Assume a region is the unit square. How many squares of length  $r = 1/10$  do we need to cover the square? If we call  $n$  this number, what is  $-\log(n)/\log(r)$ ?

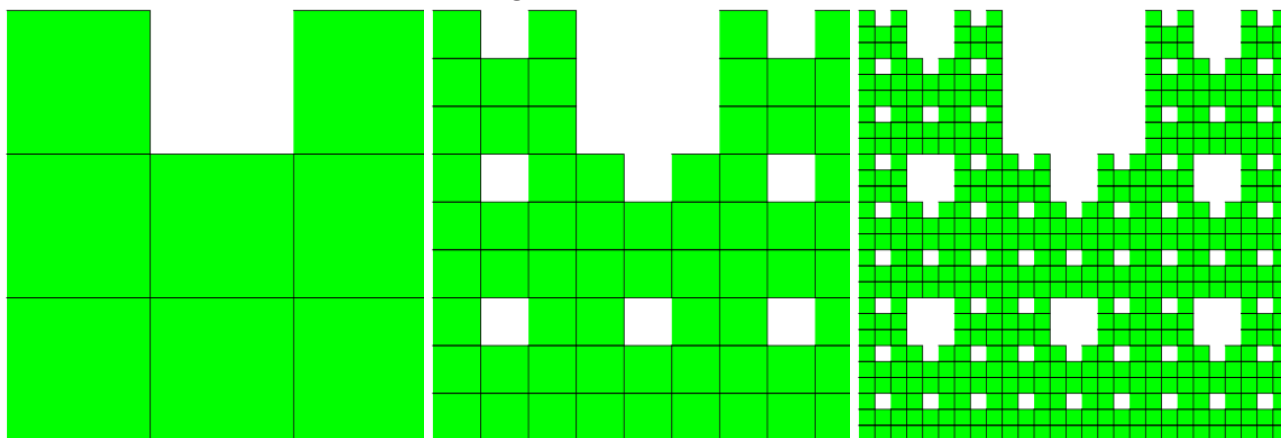
### 5. The Sirpinski carpet

The **Sirpinski carpet** is constructed recursively by dividing a square in 9 equal squares and cutting away the middle one, repeating this procedure with each of the squares etc. At the  $k$ 'th step, we need  $n = 8^k$  squares of length  $r = 1/3^k$  to cover the carpet. What is the dimension?



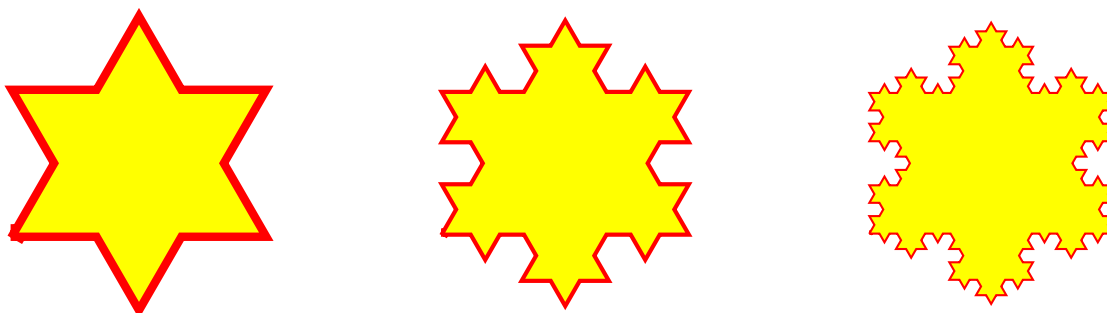

## 6. A castle

What is the dimension of the following fractal from which we see the first levels of construction?




## 7. The Koch Snowflake

What is the dimension of the Koch snowflake? How large is  $n$ , the number of squares we need to cover the flake if the square has size  $1/3^k$  assuming that the first triangle has side length 1.



## Lecture 10: Julia and Mandelbrot set

### 1. Objective

We want to understand the definition of the Julia sets and the Mandelbrot set. Define

$$T(z) = z^2 + c,$$

where  $c$  is a fixed parameter. The **filled in Julia set**  $J_c$  is the set of points  $z$  for which the orbit  $z, T(z), T(T(z))\dots$  stays bounded. The **Mandelbrot set**  $M$  is the set of  $c$  for which the point 0 is in the filled in Julia set  $J_c$ . It is the set of  $c$  such that  $0, T(0) = c, T(T(0)) = c^2 + c, T(T(T(0))) = (c^2 + c)^2 + c\dots$  stays bounded. The **Julia set** finally is the boundary of the filled in Julia set.

### 2. Complex Arithmetic

1) What is the square root of  $-9$ ?

2) Add  $2 + 6i$  with  $6 + 8i$ .

3) Multiply  $2 + 6i$  with  $6 + 8i$ .

4) What is the length of the complex number  $3 + i4$ ?

### 3. Drawing an orbit

5) Assume  $c = 2$ . Compute the first 3 steps of the orbit of  $z = 1$  of the quadratic map.

### 4. Drawing the Mandelbrot set

6) Verify that 0 is inside the Mandelbrot set. Verify that  $-1$  is inside the Mandelbrot set. Verify that  $i$  is inside the Mandelbrot set.

7) Verify that 2 is outside the Mandelbrot set. Verify that 1 is outside the Mandelbrot set.

8) Can you verify that  $1/4$  is the largest real number in the Mandelbrot set and  $-2$  the smallest real number in the Mandelbrot set?

