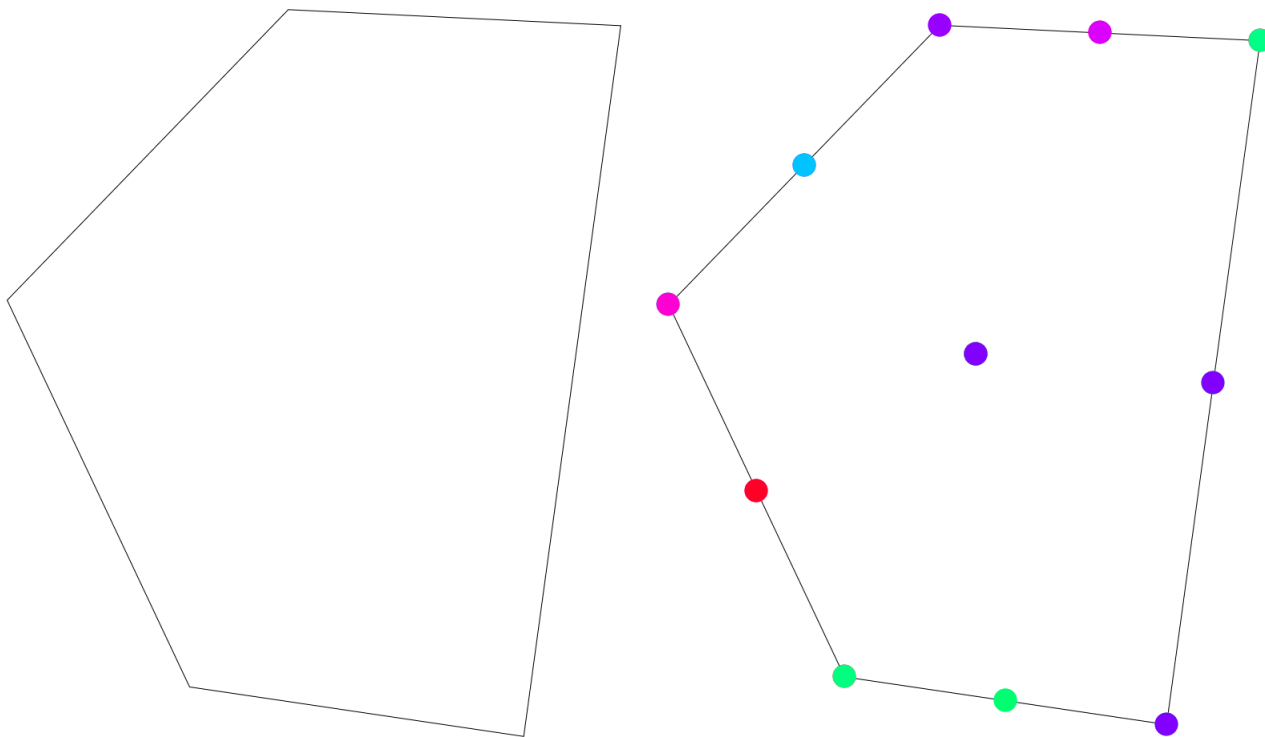


## Lecture 1: A drawing game

In this first lecture, we look a mathematical concept and try to place into the realm of mathematics. It is a topic which I stumbled upon when looking at graphs and which I still work on. The underlying construction is something which kids can already do, in order to produce pretty coloring shapes.

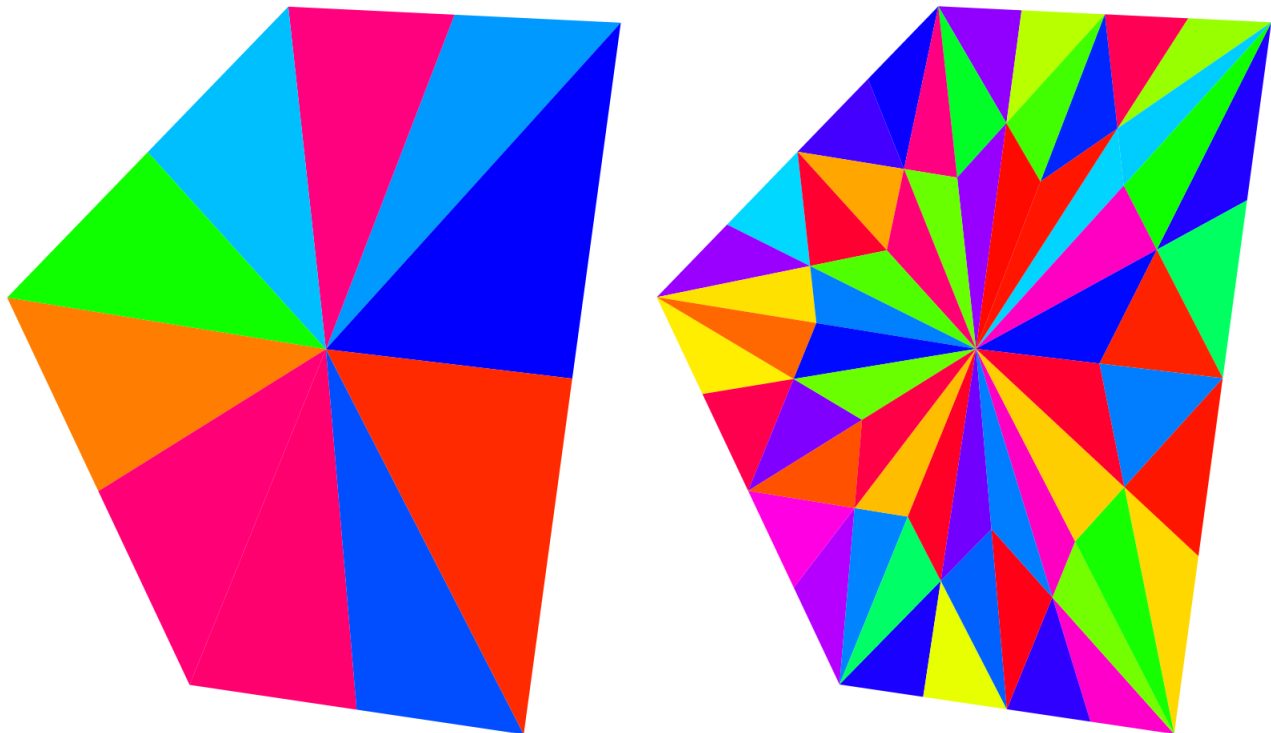


Draw a polygon onto a piece of paper. Lets assume we have drawn a pentagon. This polygon consists of  $v = 5$  vertices,  $e = 5$  edges and  $f = 1$  faces.

We will refine the polygon as follows:

- 1) Mark the edge centers and face centers.
- 2) Connect the face centers with the old vertices.
- 3) Connect the face centers with the edge centers.

We have now  $v = 11$  vertices and  $e = 20$  edges and  $f = 10$  faces.



We want to see what happens if we repeat this construction.

Now it's your turn. Take a piece of paper, draw a polygon and refine it. Once you are done. Refine again.

For your information, what you played with here is called **Barycentric subdivision**. It has applications in computer graphics.

## Questions

- 1 Let's see what we can count at the various stages. Count the number of triangles. What do you observe?
- 2 Look at the number  $v - e + f$  adding the number of vertices and faces and subtracting the number of edges. What do you observe?
- 3 How does the number of neighbors change? For example, if a vertex has 4 neighbors, how many neighbors will it have in the next step?
- 4 What can you say about the size of the triangles? Especially, what happens with the area? Can you say something about the angles?