

## Lecture 8: Probability theory

Probability theory started

### Petersburg Paradox

**Problem** We throw dices until tail appears. If  $k$  times head came first, we get  $2^k$  dollars. What is a fair fee to pay each time?

The paradox is that nobody would want to buy a 20 dollar entry fee. Mathematically, you win with probability  $2^{-k}$  an amount of  $2^k$  leading to an infinite expectation. One solution is to look what happens if there is a limit  $K$  to the jackpot. The expected win is now about  $\log_2(K)$ .

Here is a related but more primitive problem. Why does this "Martingale strategy" not work?

**Problem** We play roulette. Whenever we lose we double our input. We stop the first time we win.

Game 1: we enter a dollar and win. We stop playing.  
 Game 2: we enter a dollar and lose first. We bet 2 dollars. We lose again. We bet 4 dollars. Now we win. We have entered 7 dollars and won 8. We walk away with one dollar.



### Playing Blackjack

**Problem** Is there an optimal strategy for blackjack?

There are many variations of the game. Depending on the game, there are tables telling in which situation to hit for a new card. Here is an example.

		<< Dealers Up Card >>									
		2	3	4	5	6	7	8	9	10	A
Y o u r H a n d	8 or less	H	H	H	H	H	H	H	H	H	H
	9	H	D	D	D	D	H	H	H	H	H
	10	D	D	D	D	D	D	D	D	D	H
	11	D	D	D	D	D	D	D	D	D	H
	12	H	H	S	S	S	S	H	H	H	H
	13	S	S	S	S	S	S	H	H	H	H
	14	S	S	S	S	S	S	H	H	H	H
	15	S	S	S	S	S	S	H	H	HSU	HSU
	16	S	S	S	S	S	S	H	H	HSU	HSU
	17*	S	S	S	S	S	S	S	S	S	S
	A8-10	S	S	S	S	S	S	S	S	S	S
	A7	S	D	D	D	D	S	S	H	H	H
	A6	H	D	D	D	D	D	H	H	H	H
	A5	H	H	D	D	D	D	H	H	H	H
	A4	H	H	D	D	D	D	H	H	H	H
	A3	H	H	H	D	D	D	H	H	H	H
	A2	H	H	H	D	D	D	H	H	H	H
A-A,8-8	SP	SP	SP	SP	SP	SP	SP	SP	SP	SP	
10-10	S	S	S	S	S	S	S	S	S	S	
9-9	SP	SP	SP	SP	SP	SP	SP	SP	SP	SP	
7-7	SP	SP	SP	SP	SP	SP	SP	H	H	H	
6-6	H	SP	SP	SP	SP	SP	H	H	H	H	
5-5	D	D	D	D	D	D	D	D	D	H	
4-4	H	H	H	H	H	H	H	H	H	H	
3-3,2-2	H	H	SP	SP	SP	SP	H	H	H	H	
		2	3	4	5	6	7	8	9	10	A

<< Dealers Up Card >>

H - Hit    S - Stand    D - Double Down    SP - Split

HSU - Surrender if able to, otherwise Hit

By looking at the history of the cards played (add the low and subtract the high cards), one can increase the odds.

### Playing the Lottery

**Problem** What is the chance to hit 6 right in a lottery, where we chose 6 balls from 40?

$$\frac{6!34!}{40!} = 1/3838380$$

This is about 1 in 4 million.

There are variants of this. For Power Ball, we chose 5 from 49 and additionally one of 42 (thats the powerball). Are the odds better than for lotto?

$$\frac{5!44!}{49!42} = 1/80080128$$

The chance is one to 80 million, but there is also much more to win.