Mathematics E-320, Spring,2013

Lecture 10
Analysis

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What is Analysis?

- The Science of measure and estimation

INCLUDES

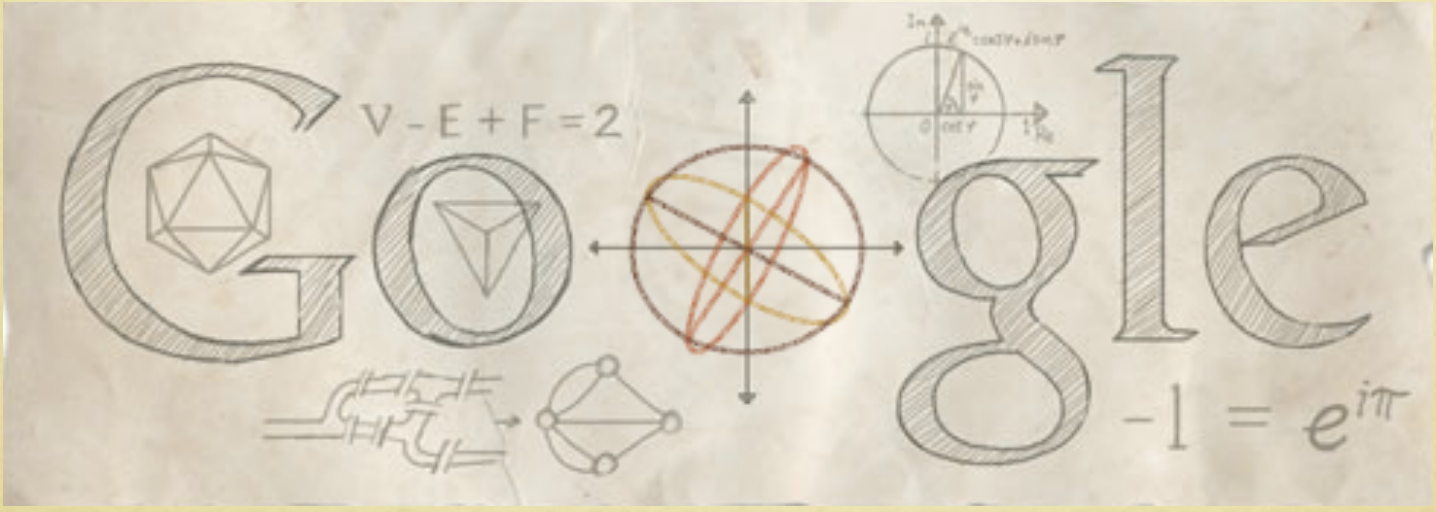
Fractal Geometry
Fourier Theory
Calculus Of Variations
Spectral Theory
Functional Analysis

Topics in Analysis

- Partial differential Equations — Weather
- Fourier Synthesis — Sound, Light
- Inverse Problems — Tomography
- Functional Analysis — Quantum Mechanics
- Calculus of Variations — Optimization Problems

Euler's Birthday

Euler worked on many problems in analysis:

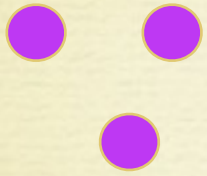


Google Doodle of Today

In order to get a sense of what Analysis is, we look at a topic, which appears in many analytic topics, the concept of Dimension and Fractal.

What is dimension?

Wish list for Dimension:

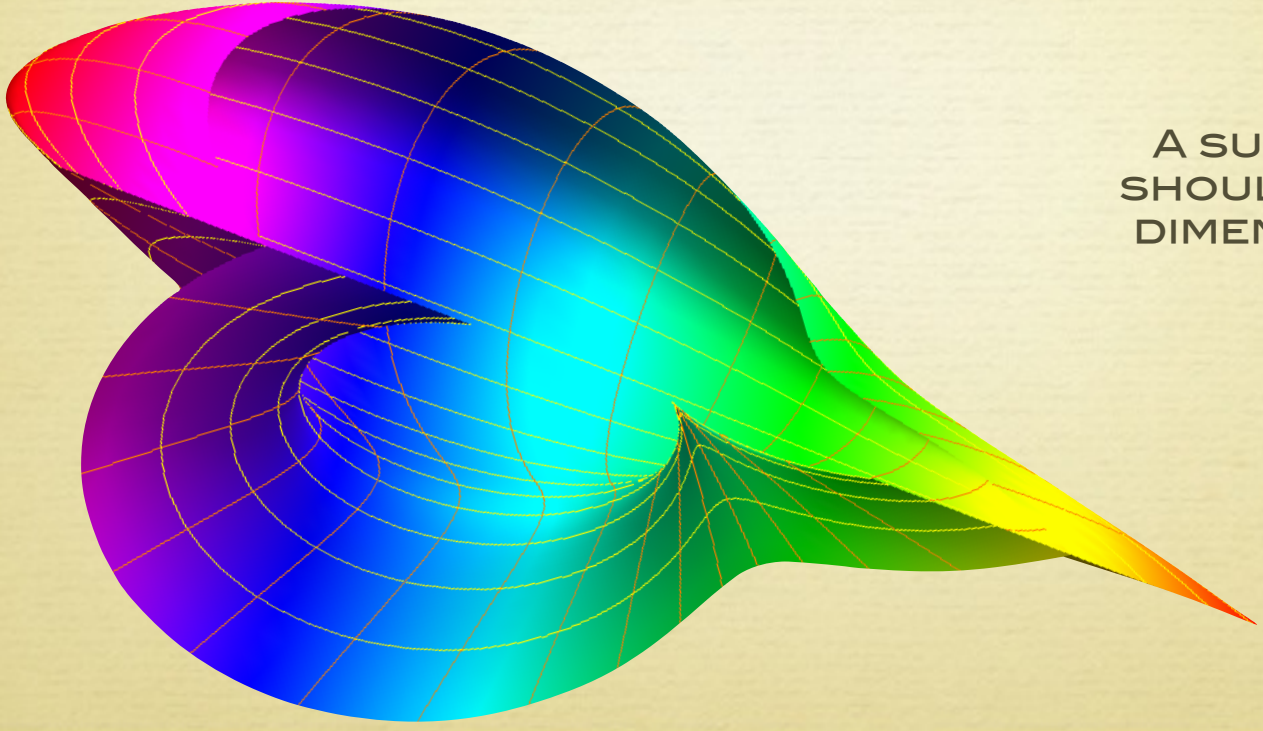


DISCRETE SETS:
DIMENSION = 0

LINE
DIMENSION = 1

PLANE
DIMENSION = 2

What is the dimension of this?



A SURFACE SHOULD HAVE DIMENSION 2

How do we measure Dimension?

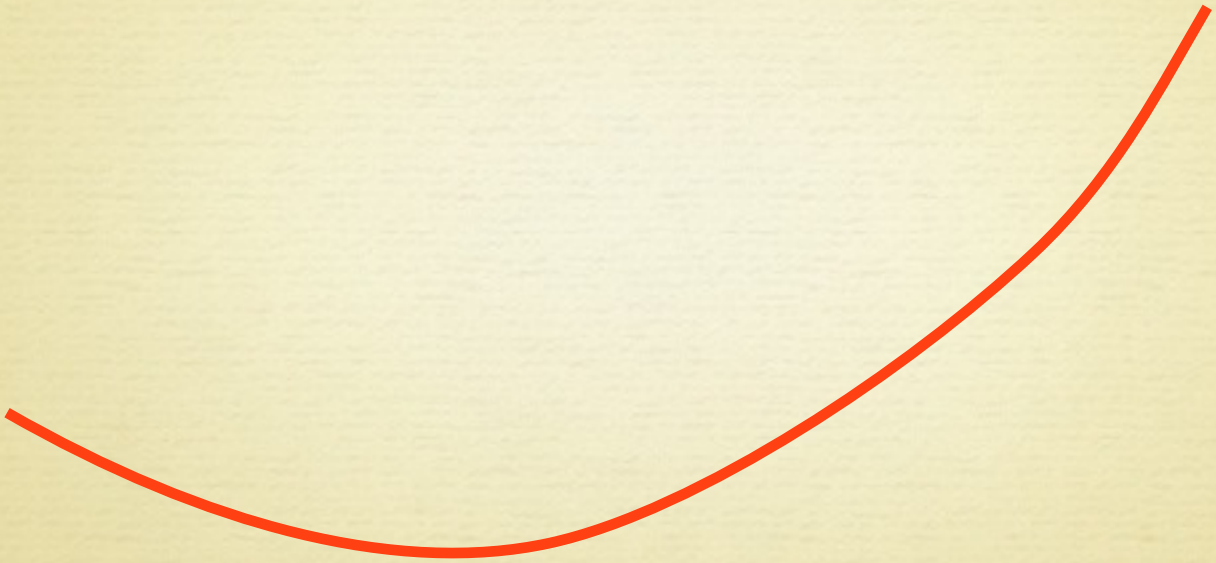
The following formula appears in the movie Star trek (2009) when

How do we measure Dimension?

The following formula appears in the movie Star trek (2009) when

$$d=-\log(n)/\log(r)$$

Dimension 1:

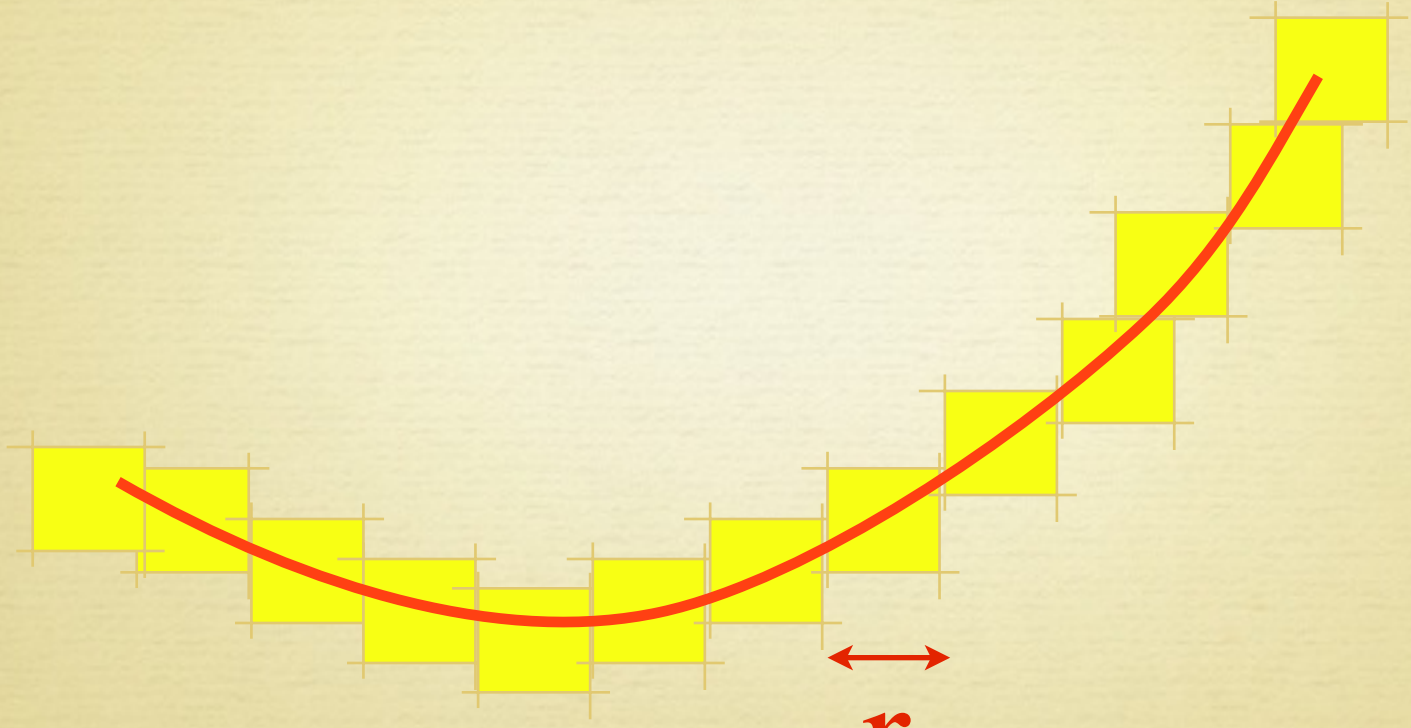


$\leftrightarrow$

r Side length

n squares

Dimension 1:

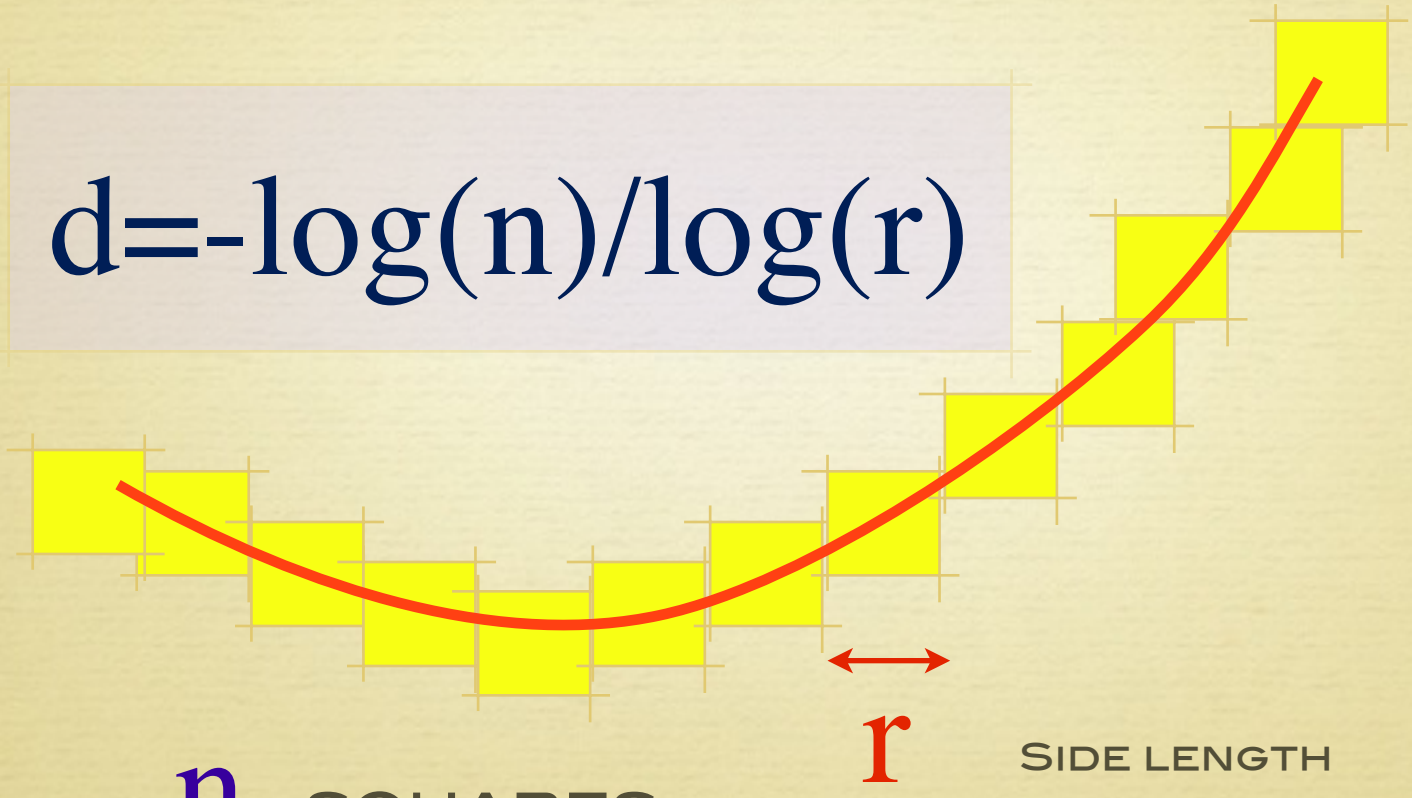


n SQUARES

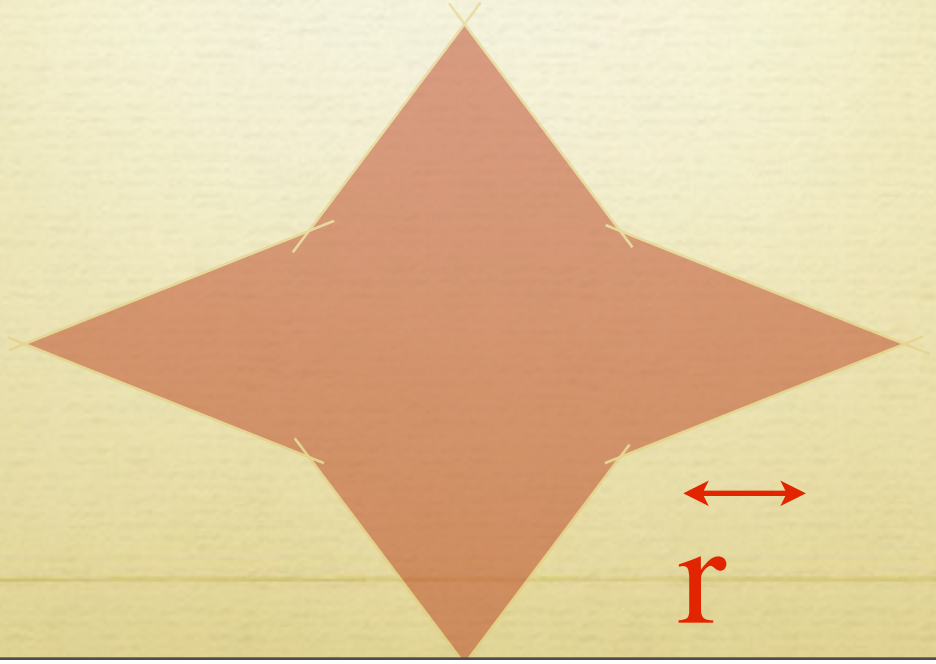
r SIDE LENGTH

Dimension 1:

$$d=-\log(n)/\log(r)$$



Dimension 2:

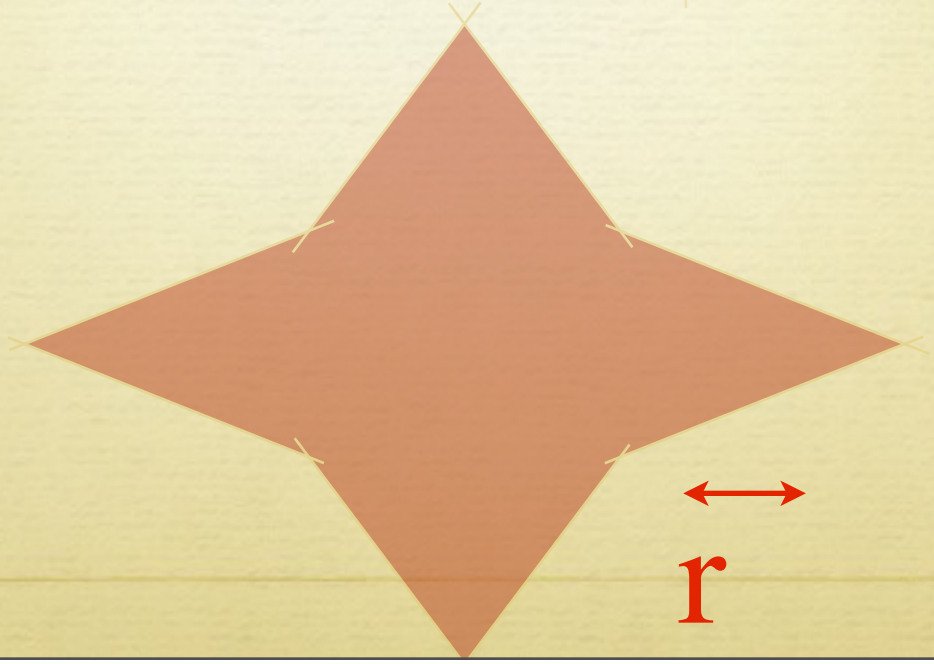


n

r

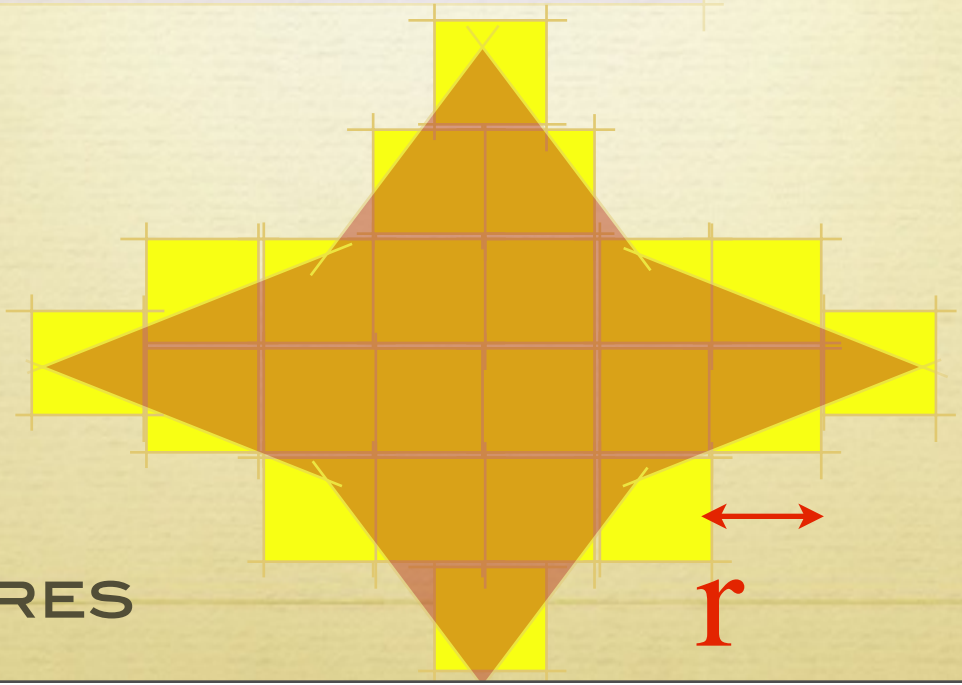
Dimension 2:

$$d=-\log(n)/\log(r)$$



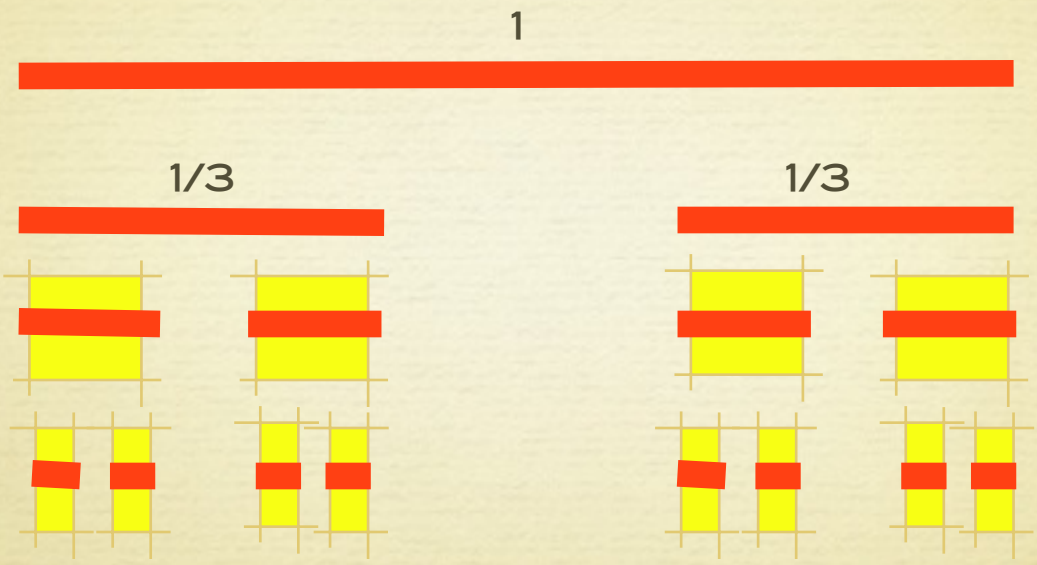
Dimension 2:

$$d=-\log(n)/\log(r)$$



n SQUARES

Cantor Set:



k=0

k=1

k=2

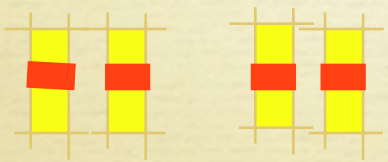
k=3

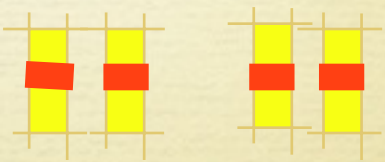
WE NEED

$$n = 2^k$$

SQUARES OF SIZE

$$r = 1/3^k$$





k=3

WE NEED

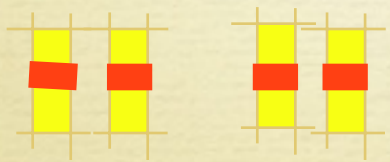
$$n = 2^k$$

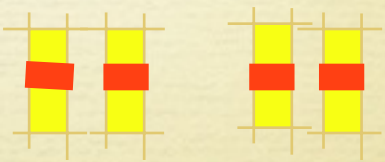
SQUARES OF SIZE

$$r = 1/3^k$$

$$d=-\log(n)/\log(r)$$

$$=\log(2)/\log(3)$$





$k=3$

WE NEED

$$n = 2^k$$

SQUARES OF SIZE

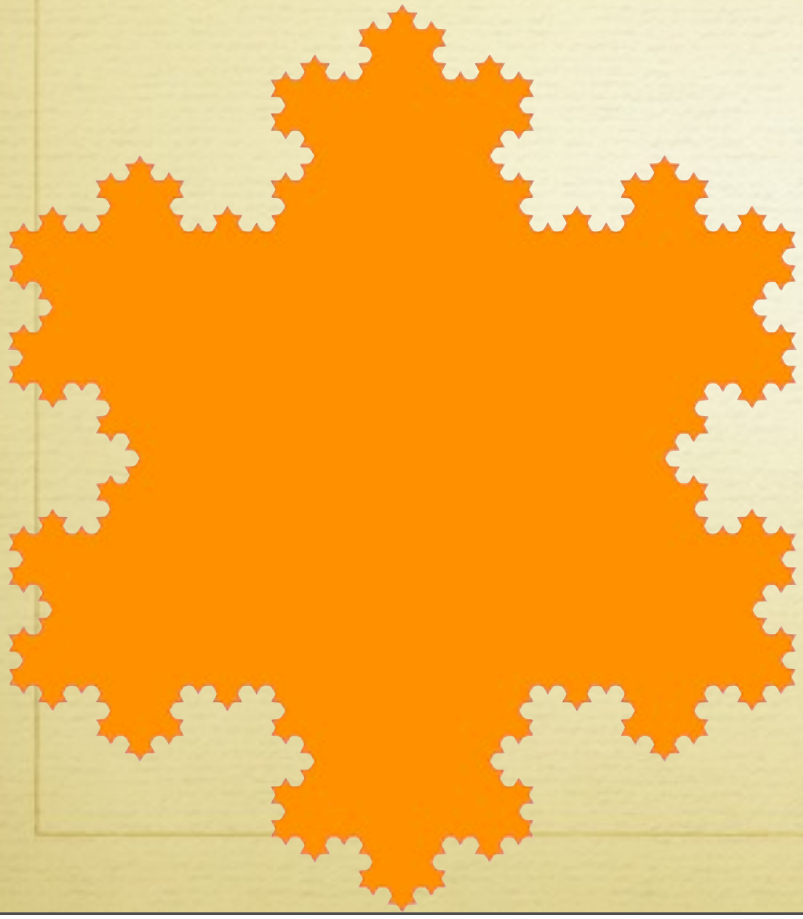
$$r=1/3^k$$

Georg Cantor, 1845-1918

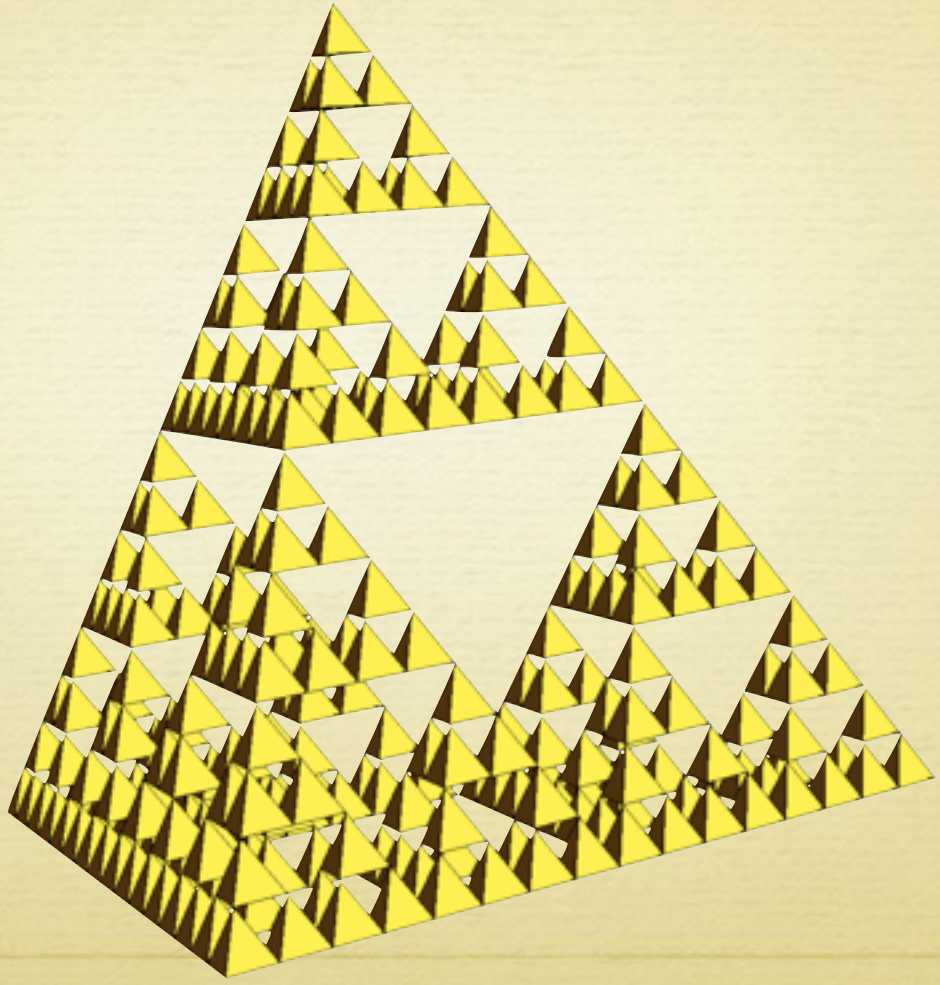
Fractals:

An object with noninteger dimension.

Koch Curve

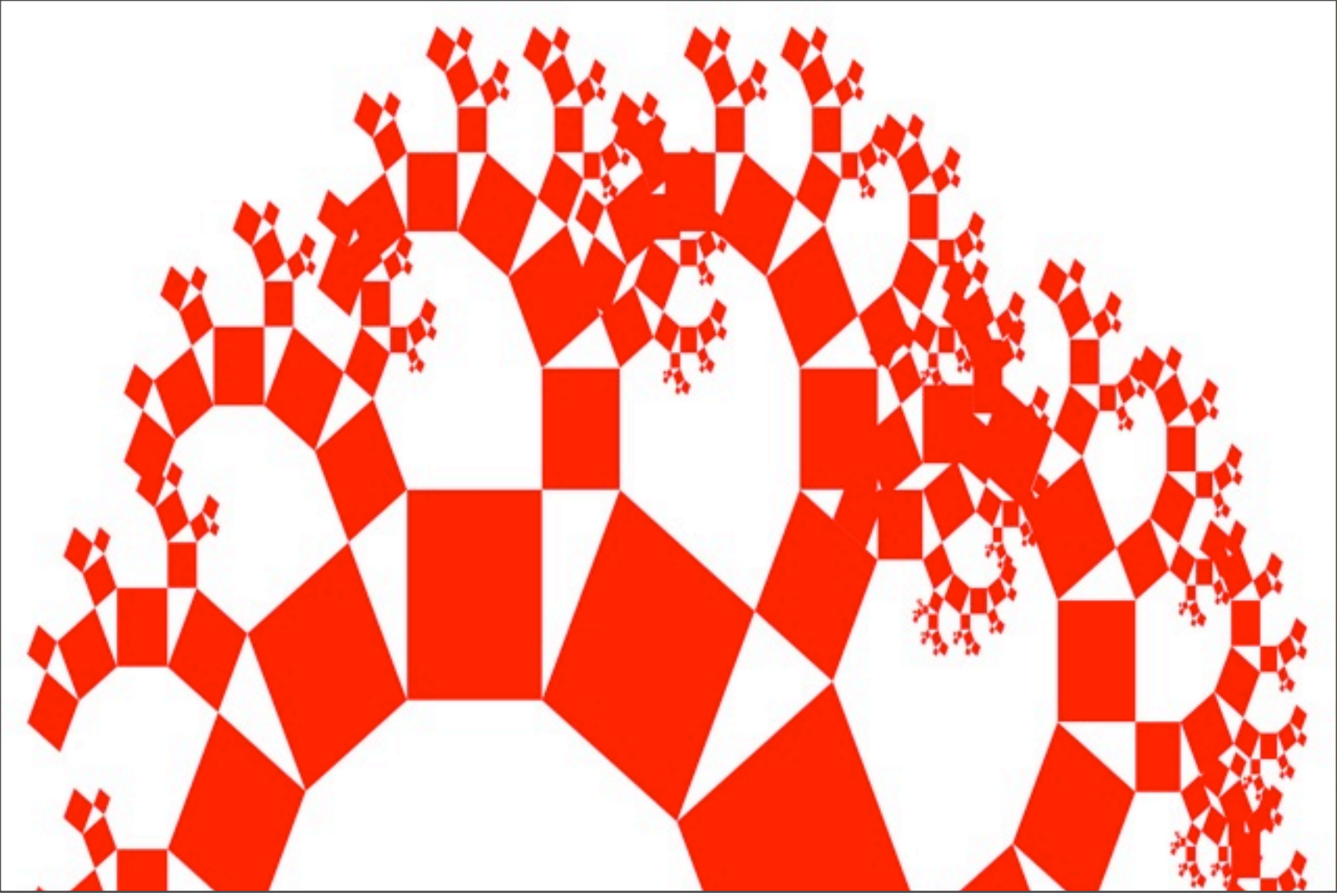


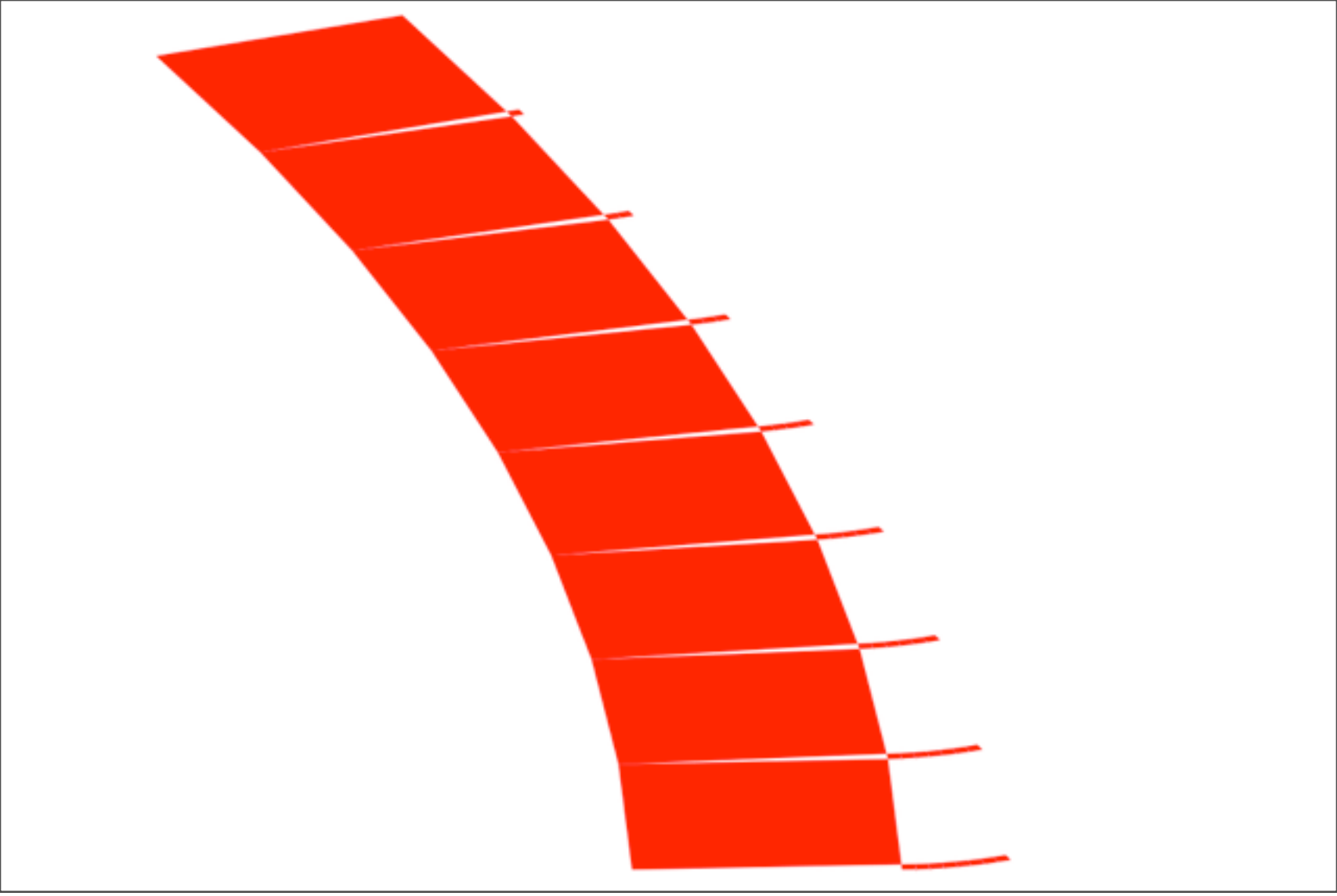
Sierpinski pyramid



Tree of Pythagoras

Albert Bosman 1942

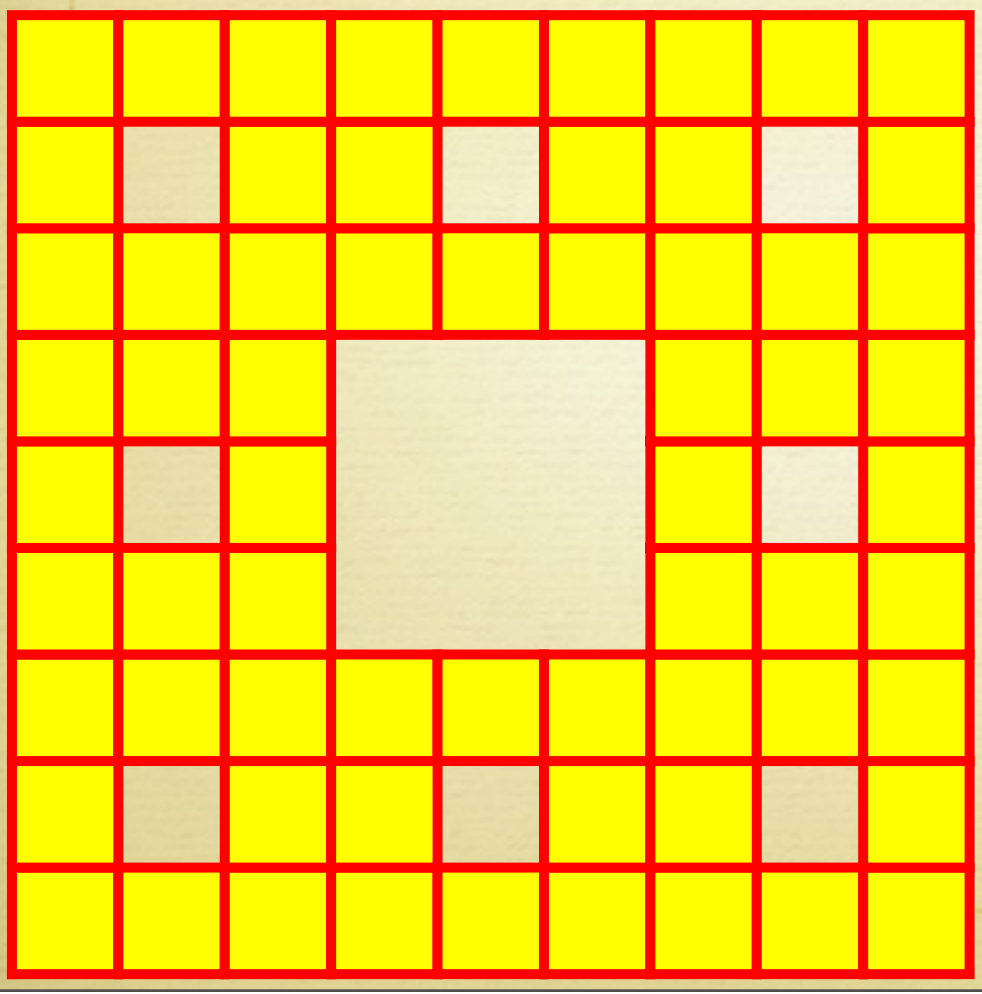




Benoît Mandelbrot

Sierpinski carpet

1916



$d=-\log(n)/\log(r)$

$N=64=8^K$

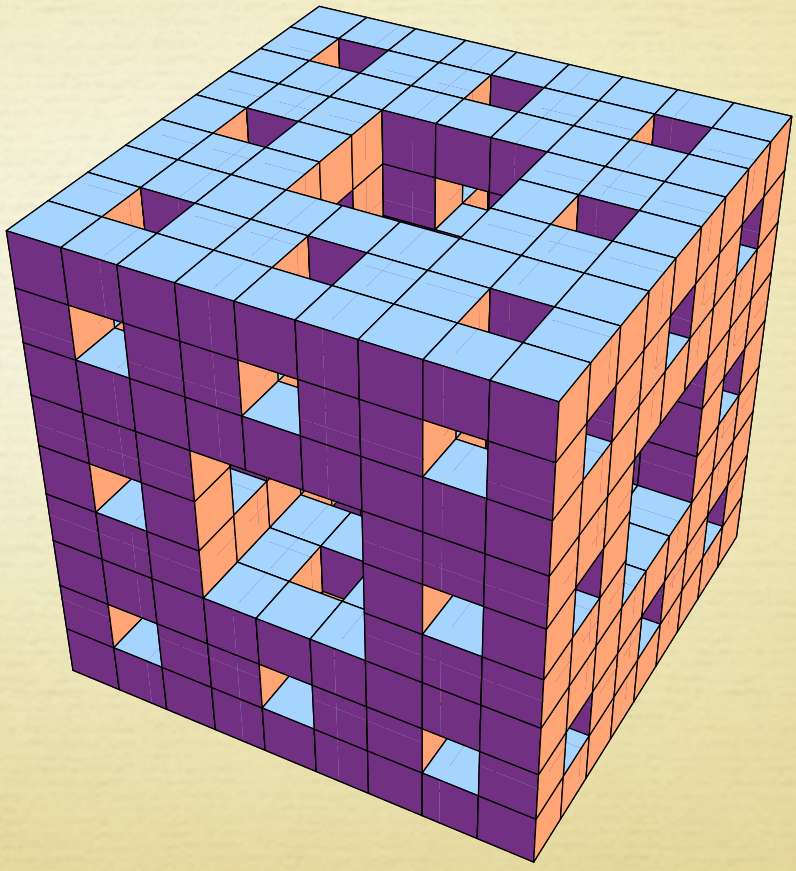
$R=1/9=1/3^K$

Limit:

$\log(8)/\log(3)$

$=1.89279$

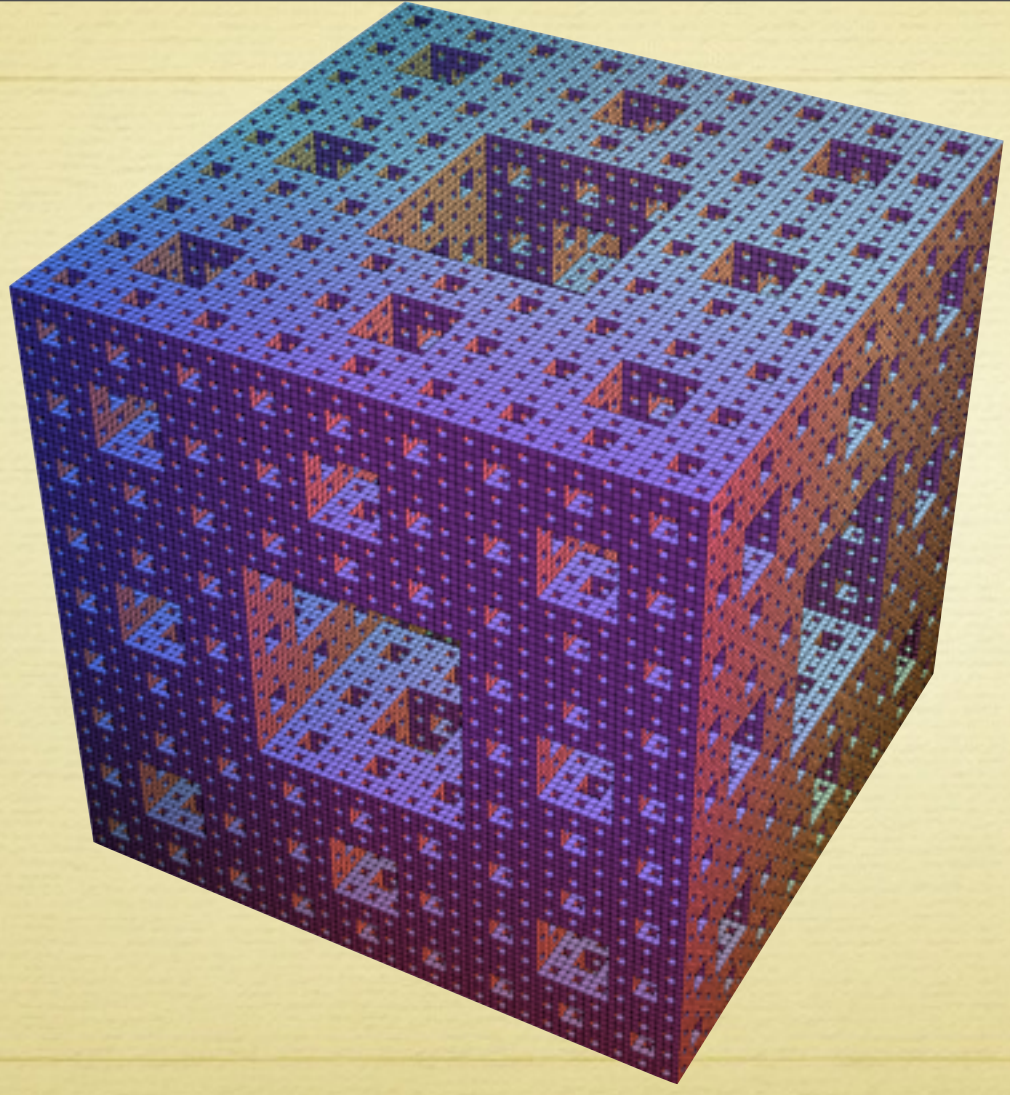
Menger Sponge



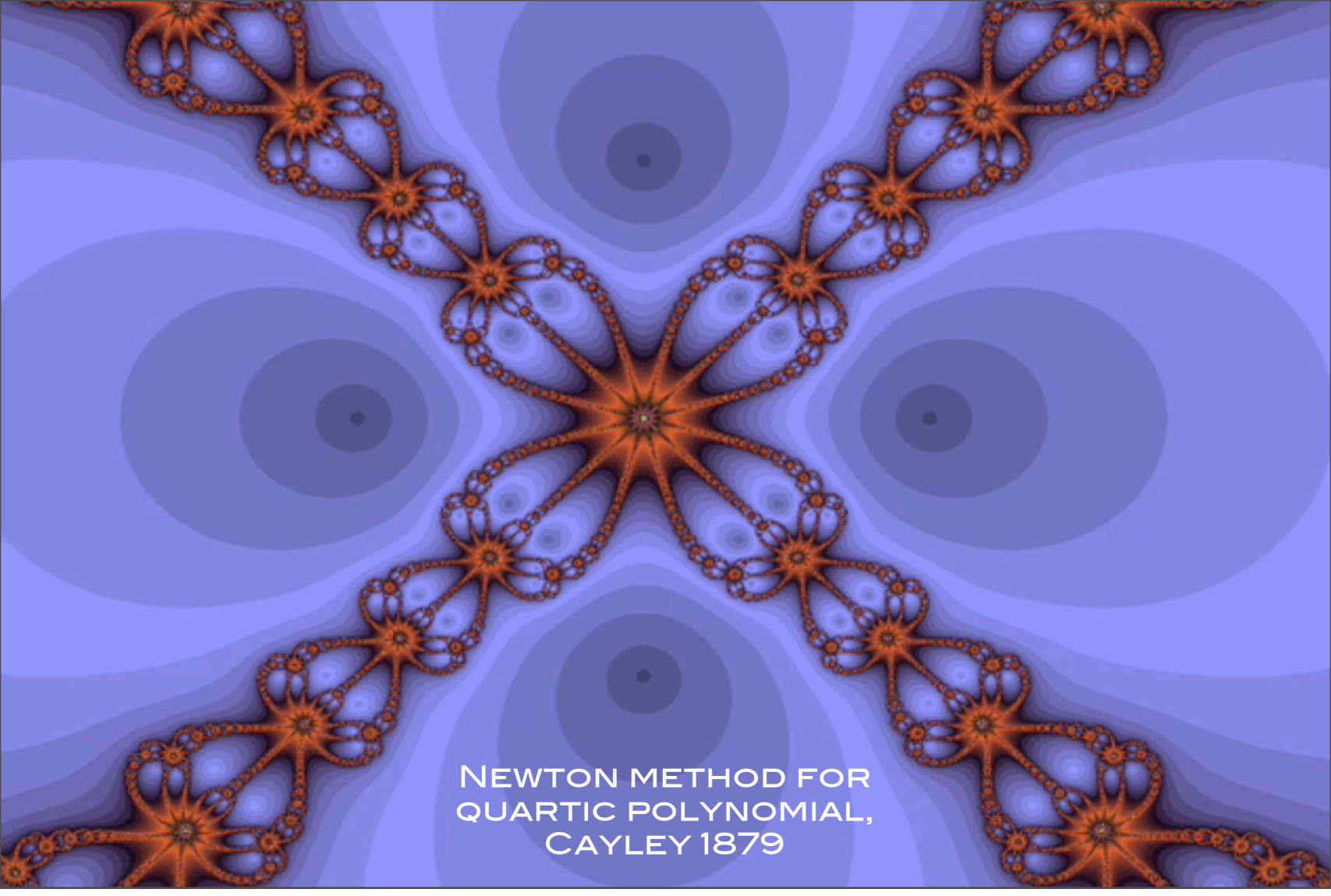
log(20)/log(3)

=2.7268...

Karl Menger: 1902-1985

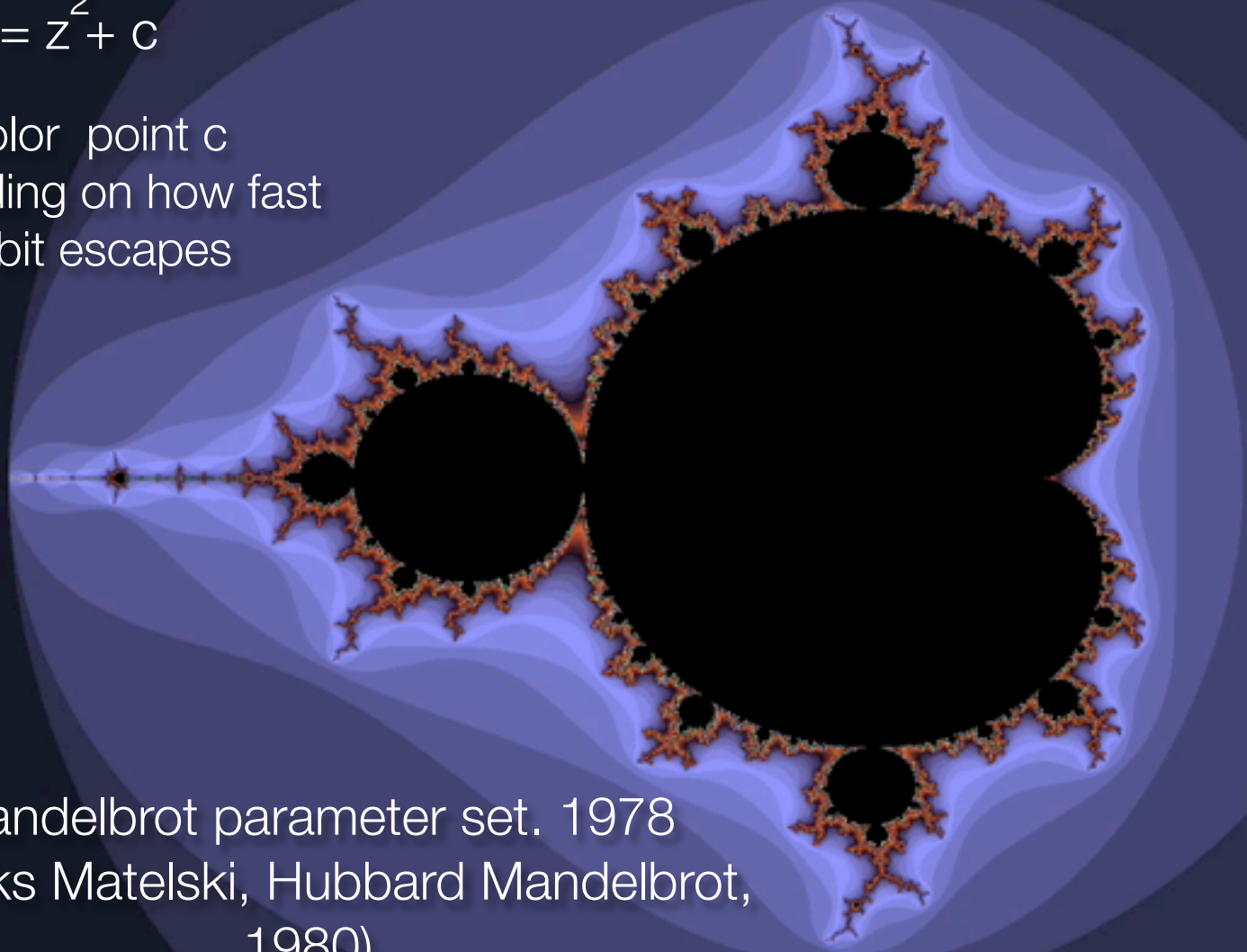


Other Fractals


Newton method for
quartic polynomial,
Cayley 1879

$T(z) = z^2 + c$

color point c
according on how fast
orbit escapes



Mandelbrot parameter set. 1978
(Brooks Matelski, Hubbard Mandelbrot,
1980)

The mandelbulb

A YOUTUBE STAR

How does it work?

Z -> Z^N + C IN SPHERICAL COORDINATES.

Daniel White 2007, Inspired by Rudy Rucker

Literature

- Barnsley: fractals everywhere
- Mandelbrot: Geometry of Fractals