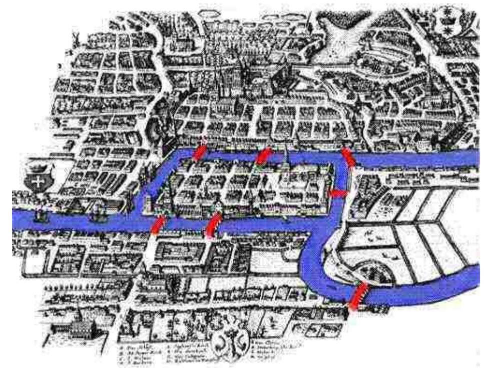
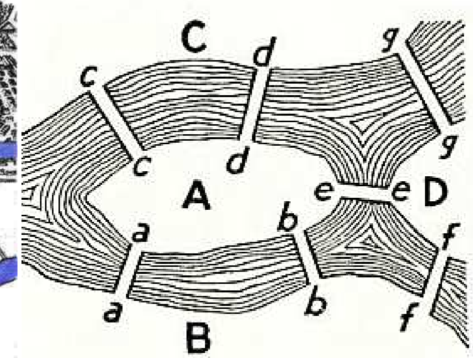
Lecture 9: Topology

In topology, objects which can be deformed into each other are the same. For a topologist, all triangles are the same. A triangle and a circle are the same too. They can be deformed into each other.

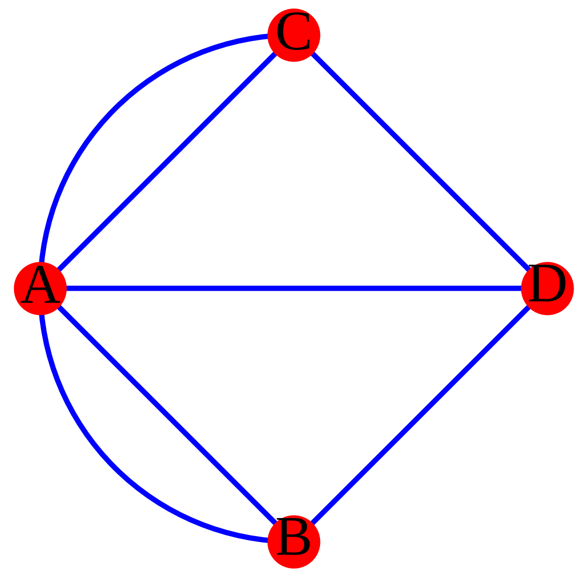
The Boston subway map for example is a topologically equivalent representation of the real subway paths. But the lengths are not on scale. This makes it easier to read.

One of the starting points of topology is the Königsberg bridge problem. Is it possible to find a walk which which crosses every bridge once and only once?





Euler realized that one can see this as an abstract problem about graphs. It it possible find a path through the graph which covers the entire graph but no edge twice? Such a path is called an **Eulerian path**. If the start and end point is the same it is called an **Eulerian circuit**.

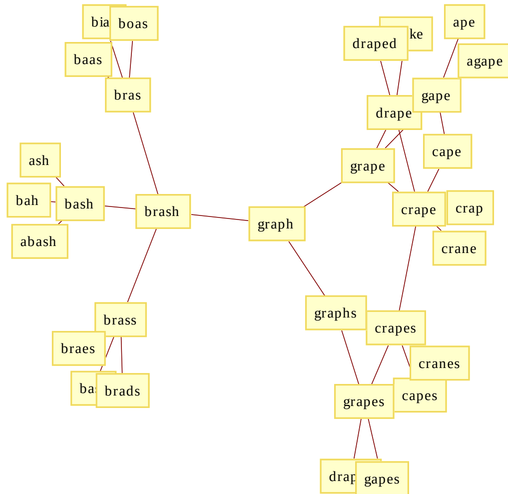


Euler presented his work in 1735. It was published as "Solutio problematis ad geometriam situs pertinentis" in 1741.

This is historically significant, because it is one of the first results in graph theory an area of mathematics closely related to topology because a lot of topology have analogue results on graphs.

There is more to that problem than just a new field of mathematics:

The problem shows how mathematical abstraction can simplify a problem.



Words in the neighborhood of "graph". **Problem 2:** Find the Euler characteristic $\chi(G) = v - e + f$, where v is the number of vertices, e the number of edges and f the number of triangles.
Problem 3: Compute the curvature $K(x) = 1 - |V(x)|/2 + |E(x)|/3$ at each point where $|V|$ is the number of edges and $E(x)$ is the number of vertices in $S(x)$. Add them up and see whether they agree with $\chi(G)$.