

ALL BOBS

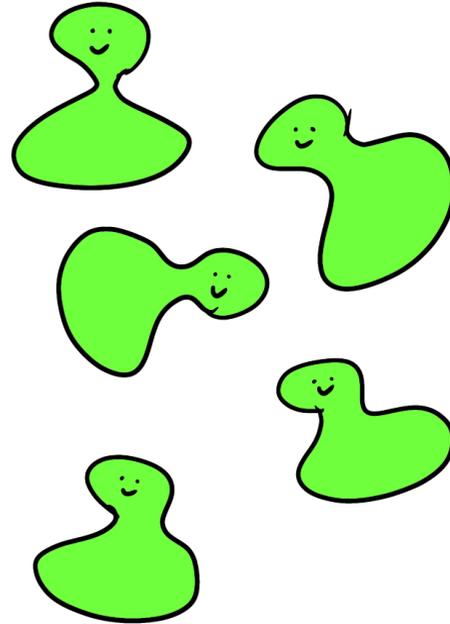
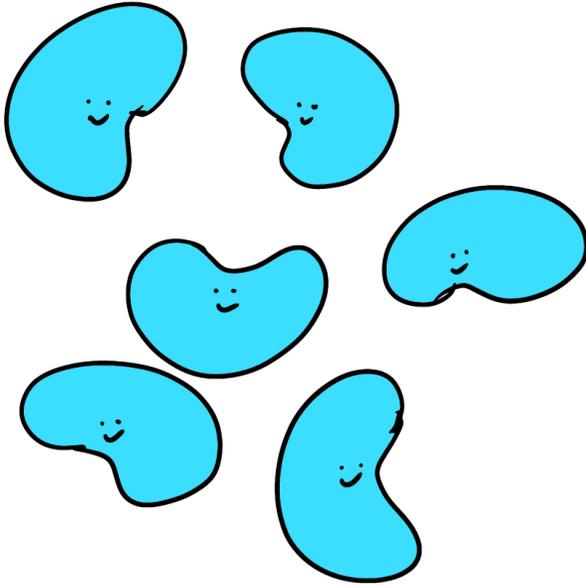
a proof book in pictures

ARE BLOBS

written & illustrated by Kieran Chung

Axiom 1: All creatures are either blobs or glogs.

Axiom 2: All blobs are blue.



Axiom 3: All glogs are green.

Axiom 4: All blue creatures have names starting with B.

I'm Bob!



I'm Brenda!



I'm Gina!



I'm Gary!



I'm Bart!

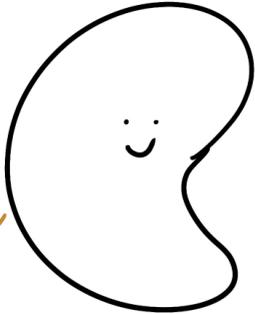


I'm Gus!

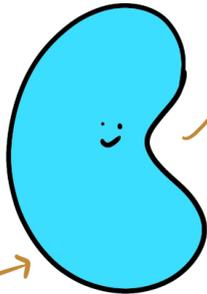
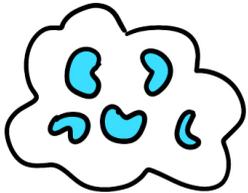
Axiom 3: All green creatures have names starting with G.

Proposition 1: All blobs have names starting with B.

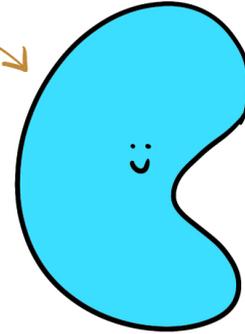
1. Suppose X is a blob.



2. Since all blobs are blue, X is blue.



3. Since all blue creatures have names starting with B, X's name starts with B.



- I'm Bob!

4. Therefore, all blobs have names starting with B.

- I'm
Brenda!



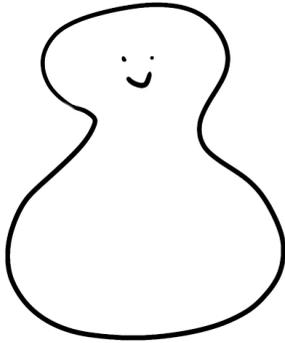
- I'm Bob!



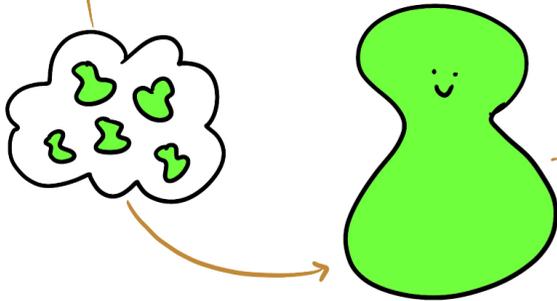
- I'm Bart!

Proposition 2: All glogs have names starting with G.

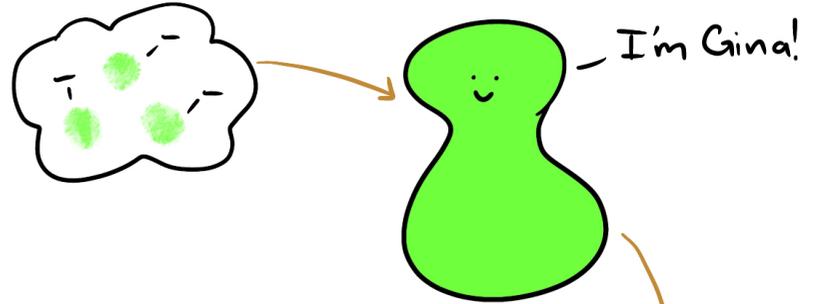
1. Suppose X is a glog.



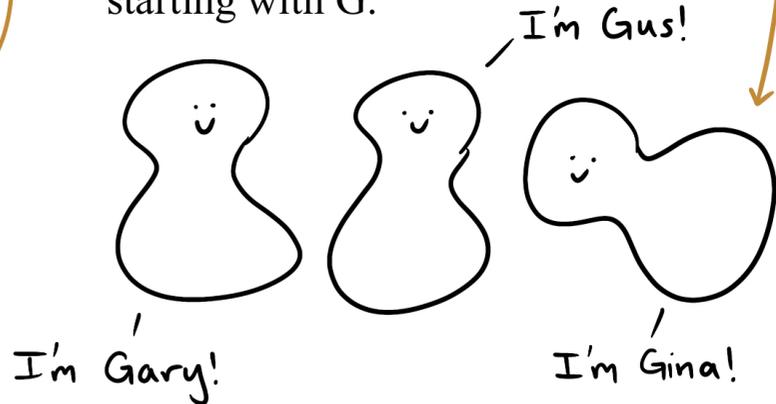
2. Since all glogs are green, X is green.



3. Since all green creatures have names starting with G, X's name starts with G.

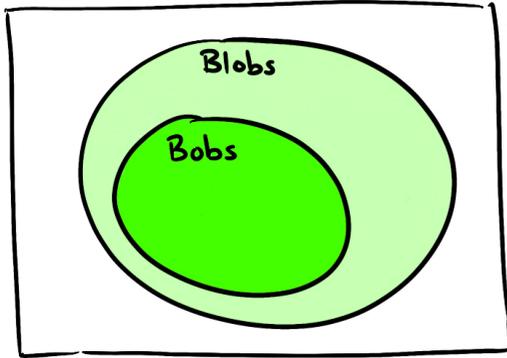


4. Therefore, all glogs have names starting with G.



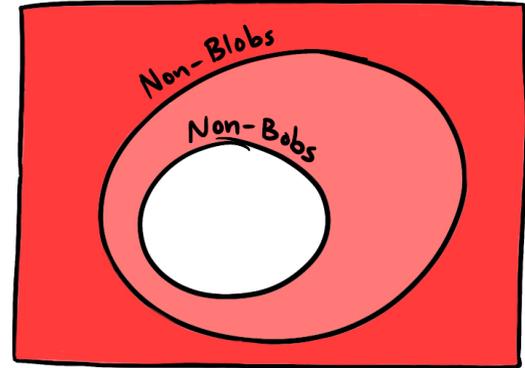
Proposition 3: All Bobs are blobs.

Tool: **Contraposition** 



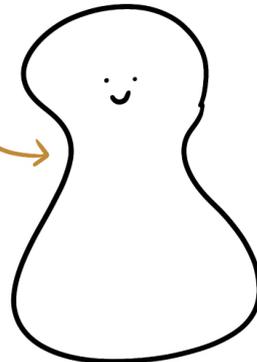
To prove all Bobs are blobs...

1. Suppose X is not a blob.

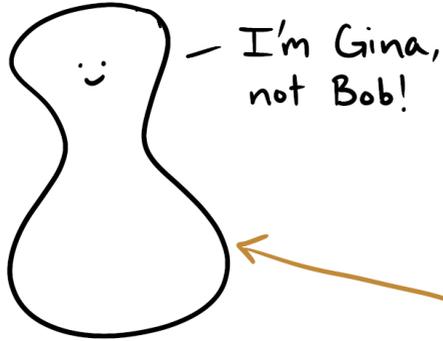


Prove all non-blobs are non-Bobs.

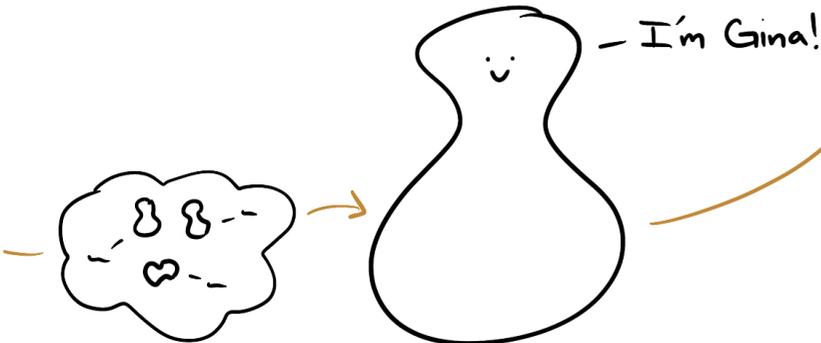
2. Since all creatures are blobs or glogs, X is a glog.



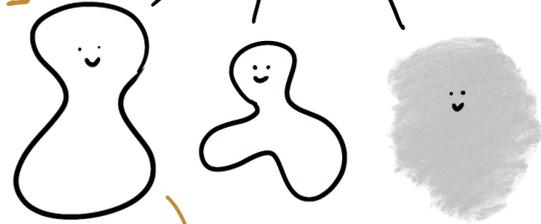
4. Since B is not G, X is not a Bob.



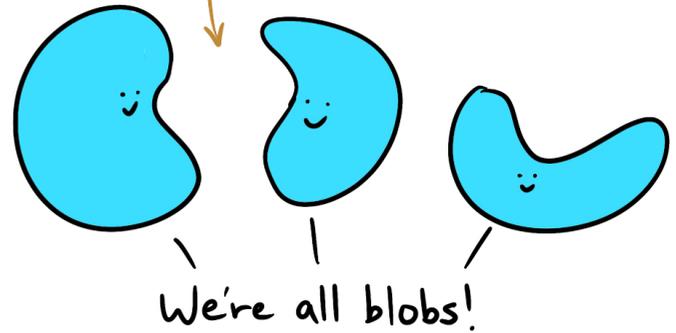
3. Since all glogs have names starting with G, X's name starts with G.



We're not Bobs!



5. Therefore, all non-blobs are non-Bobs. By **contraposition**, all Bobs are blobs.



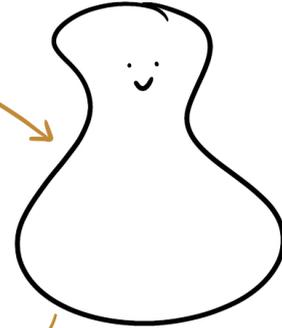
Proposition 4: All blue creatures are blobs.



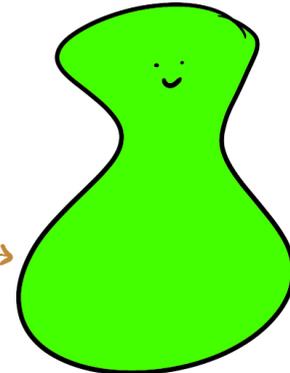
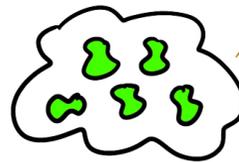
1. Suppose X is not a blob.



2. Since all creatures are blobs or glogs, X is a glog.



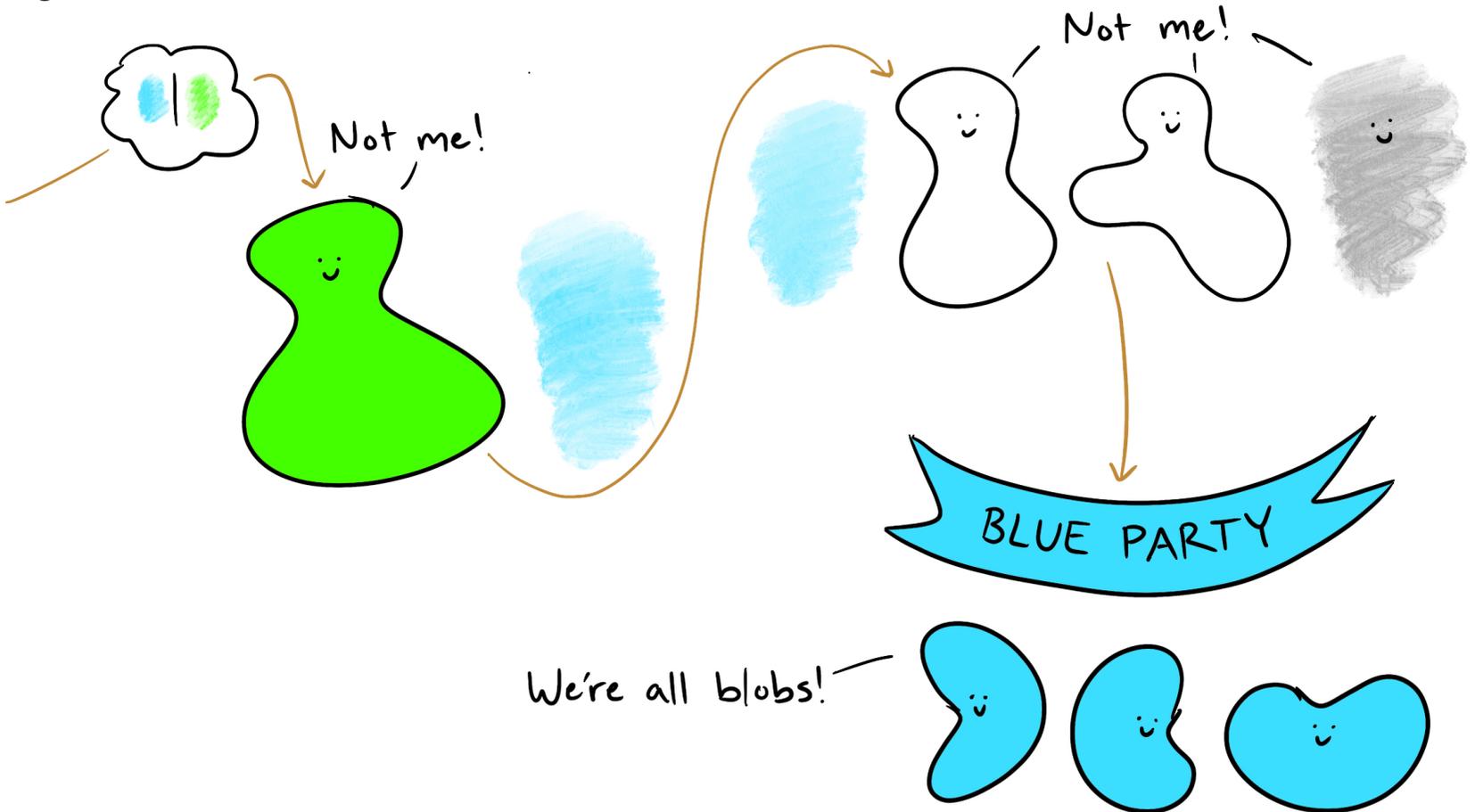
3. Since all glogs are green, X is green.



*By Proposition 6!

4. Since no creature is both blue and green*, X is not blue.

5. Therefore, by **contraposition**, all blue creatures are blobs.



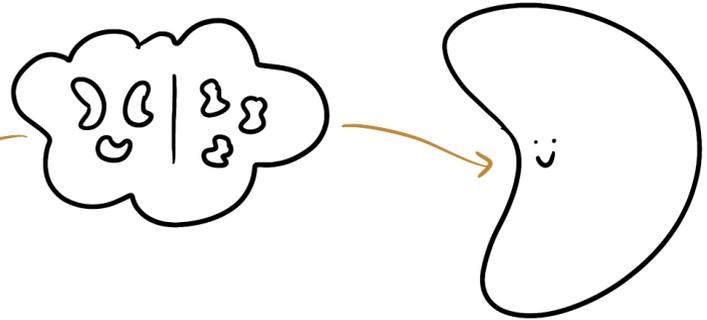
Proposition 5: All green creatures are glogs.



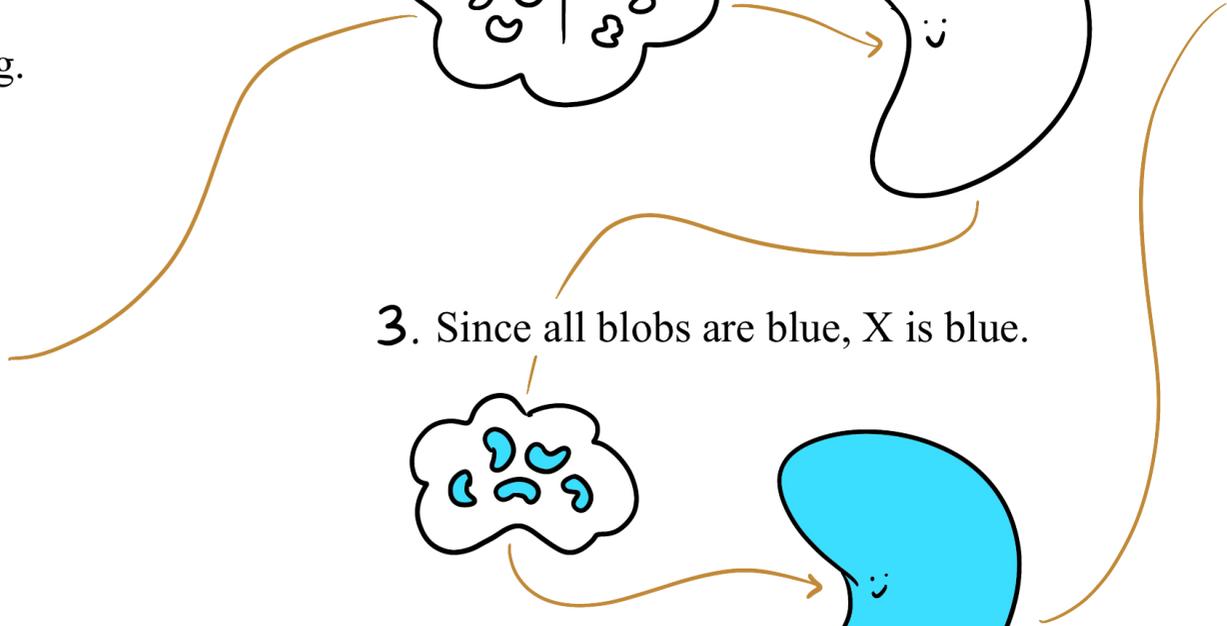
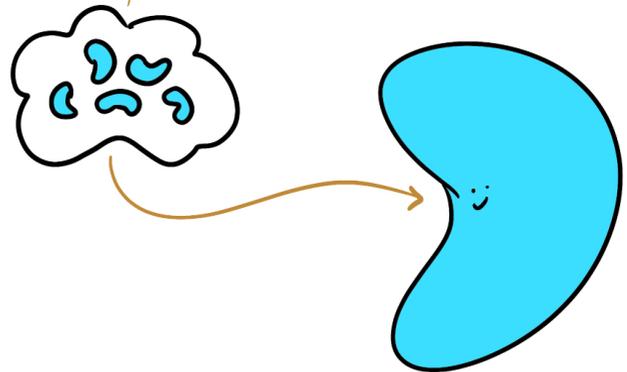
1. Suppose X is not a glog.



2. Since all creatures are blobs or glogs, X is a blob.



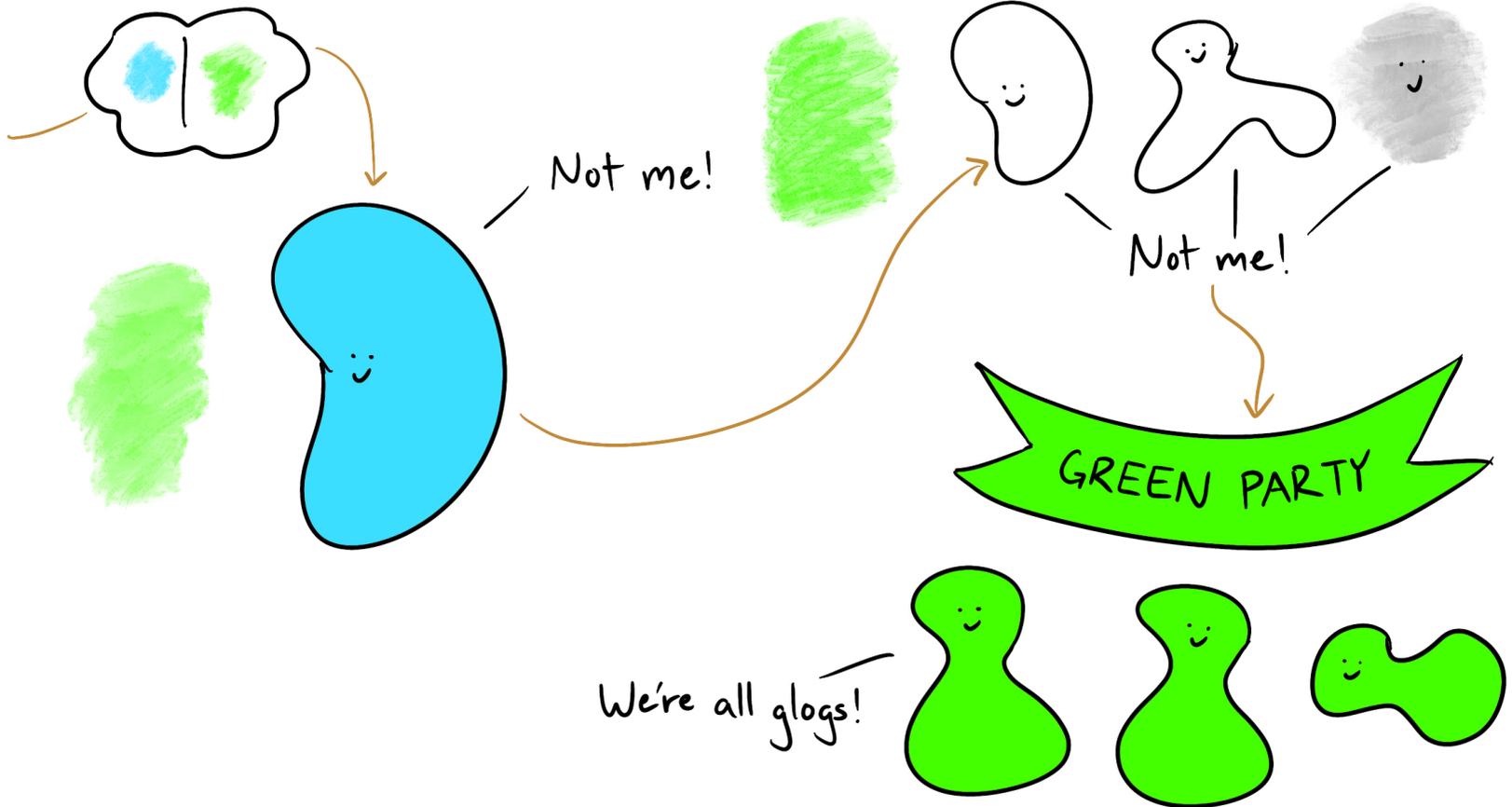
3. Since all blobs are blue, X is blue.



* By Proposition 6!

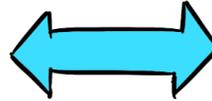
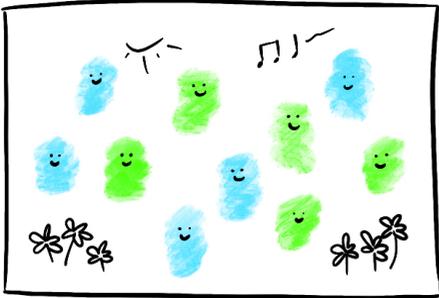
4. Since no creature is both blue and green,* X is not green.

5. Therefore, by **contraposition**, all green creatures are glogs.



Proposition 6: No creature is both blue and green.

Tool: **Contradiction** 

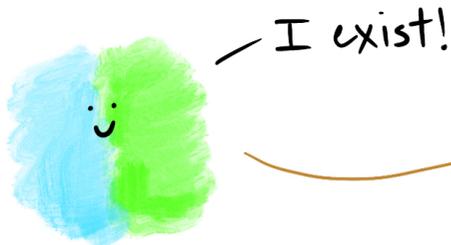


To prove no creature is both blue and green...

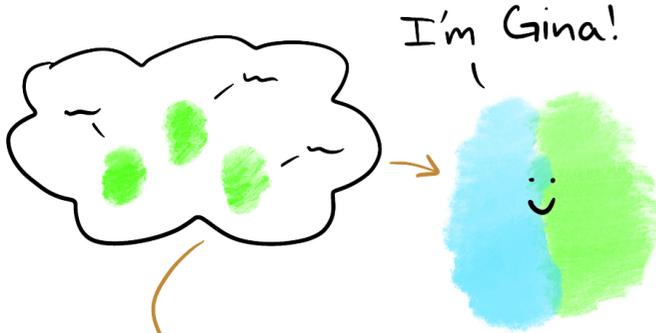
Assume one is, and show something goes wrong.

1. Assume **for the sake of contradiction** that some creatures are both blue and green.

2. Then at least one creature, X, is both blue and green.



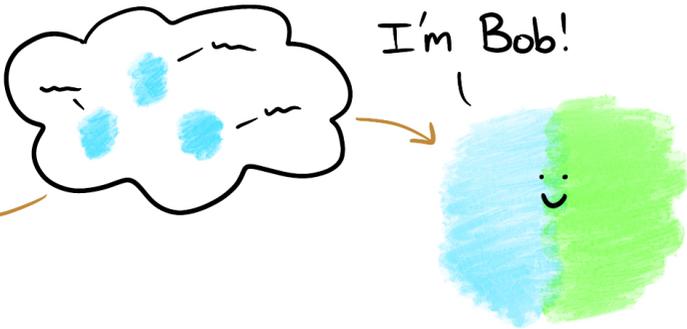
4. Since all green creatures have names starting with G, X's name starts with G.



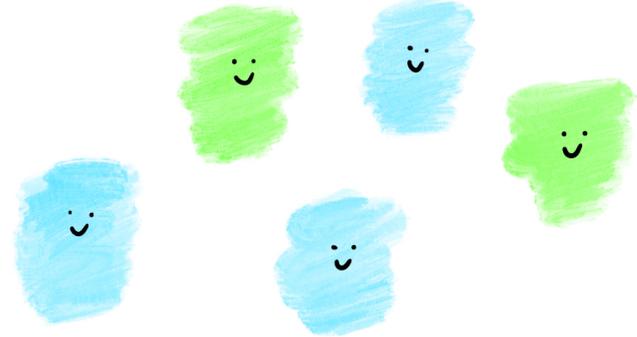
5. But since B is not G, this is a **contradiction**.



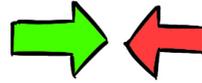
3. Since all blue creatures have names starting with B, X's name starts with B.



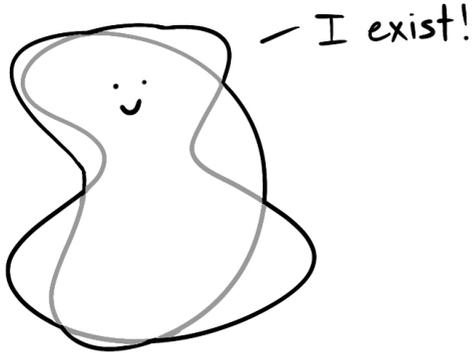
6. Therefore, no creature is both blue and green.



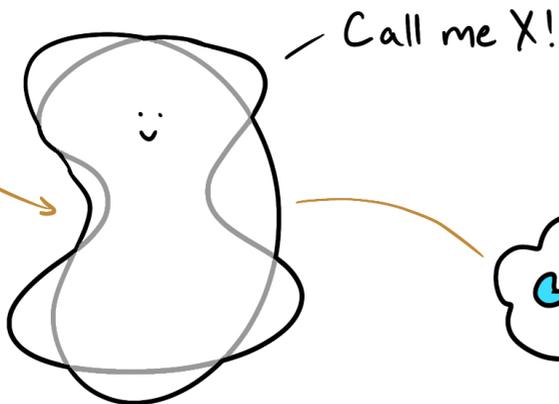
Proposition 7: No blobs are glogs.



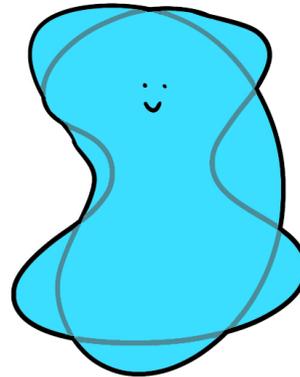
1. Assume **for the sake of contradiction** that some blobs are glogs.



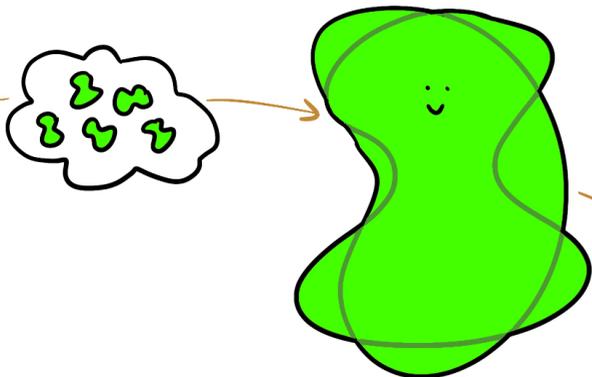
2. Then at least one blob, X, is also a glog.



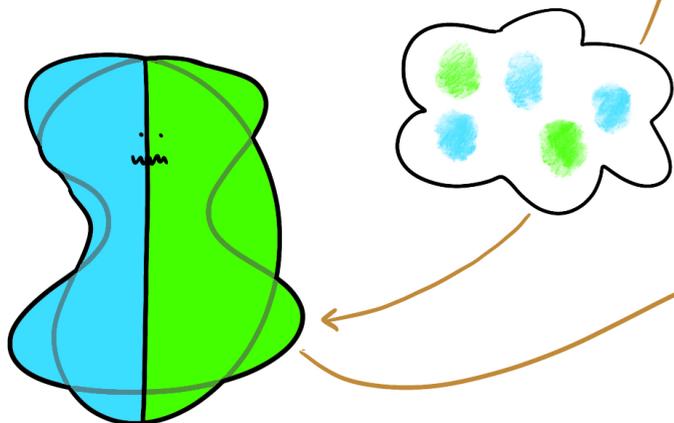
3. Since all blobs are blue, X is blue.



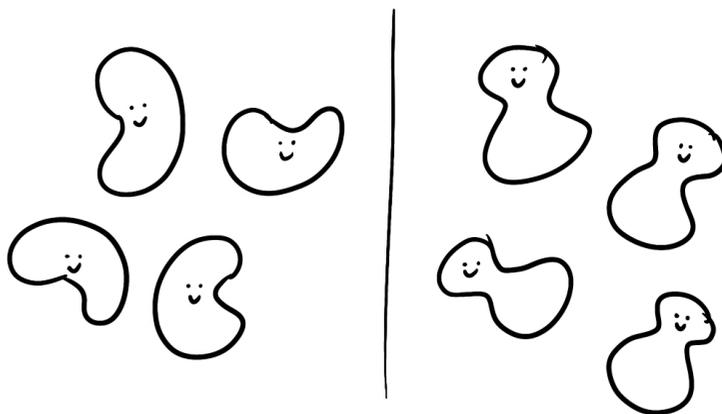
4. Since all glogs are green, X is green.



5. But since no creature is both blue and green, this is a **contradiction**.



6. Therefore, no blobs are glogs.

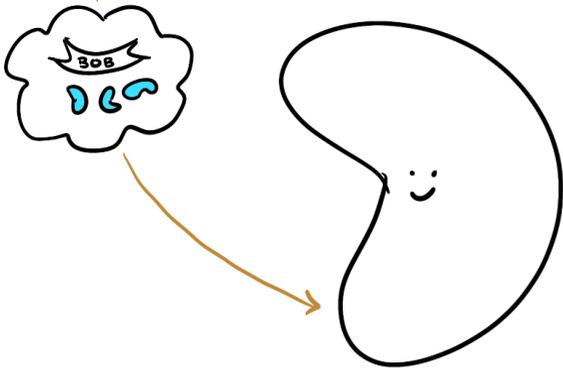


Proposition 8: No Bobs are glogs.

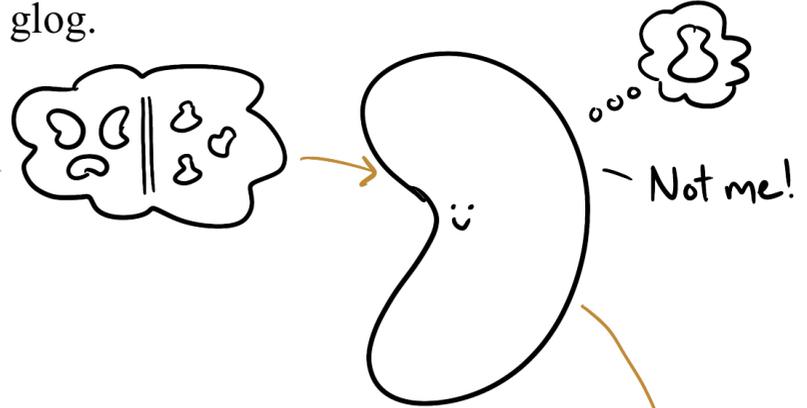
1. Suppose this is Bob.



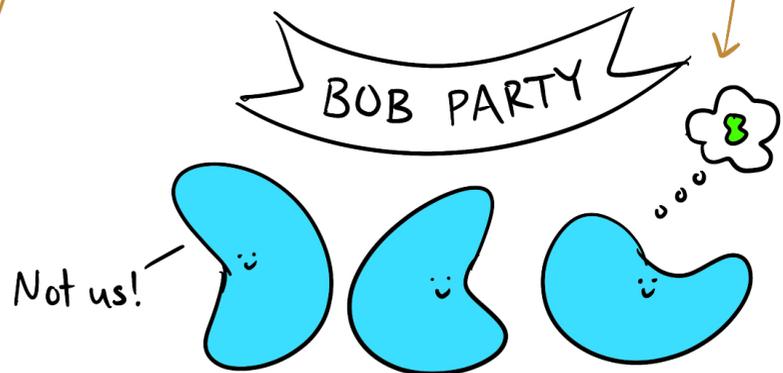
2. Since all Bobs are blobs, Bob is a blob.



3. Since no blobs are glogs, Bob is not a glog.



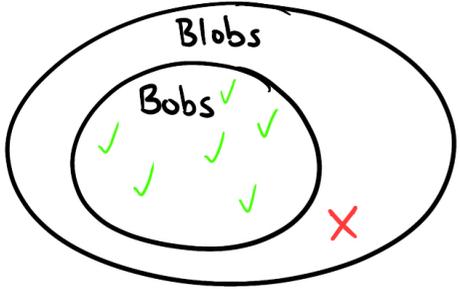
4. Therefore, no Bobs are glogs.



Proposition 9: Not all blobs are Bobs.

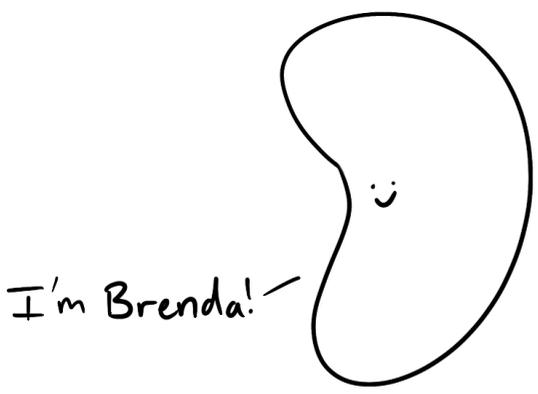
Tool: **Counterexample** 

To prove not all blobs are Bobs...

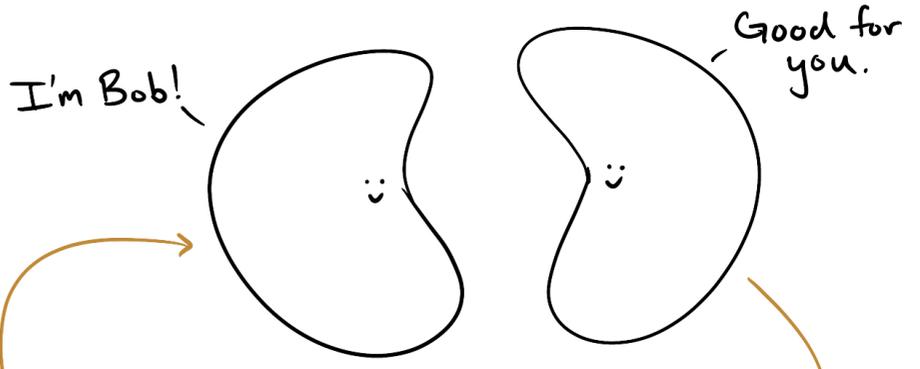


Find a blob that isn't a Bob.

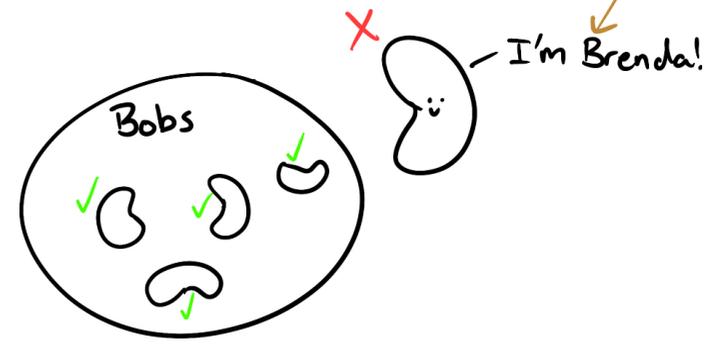
1. This is Brenda.



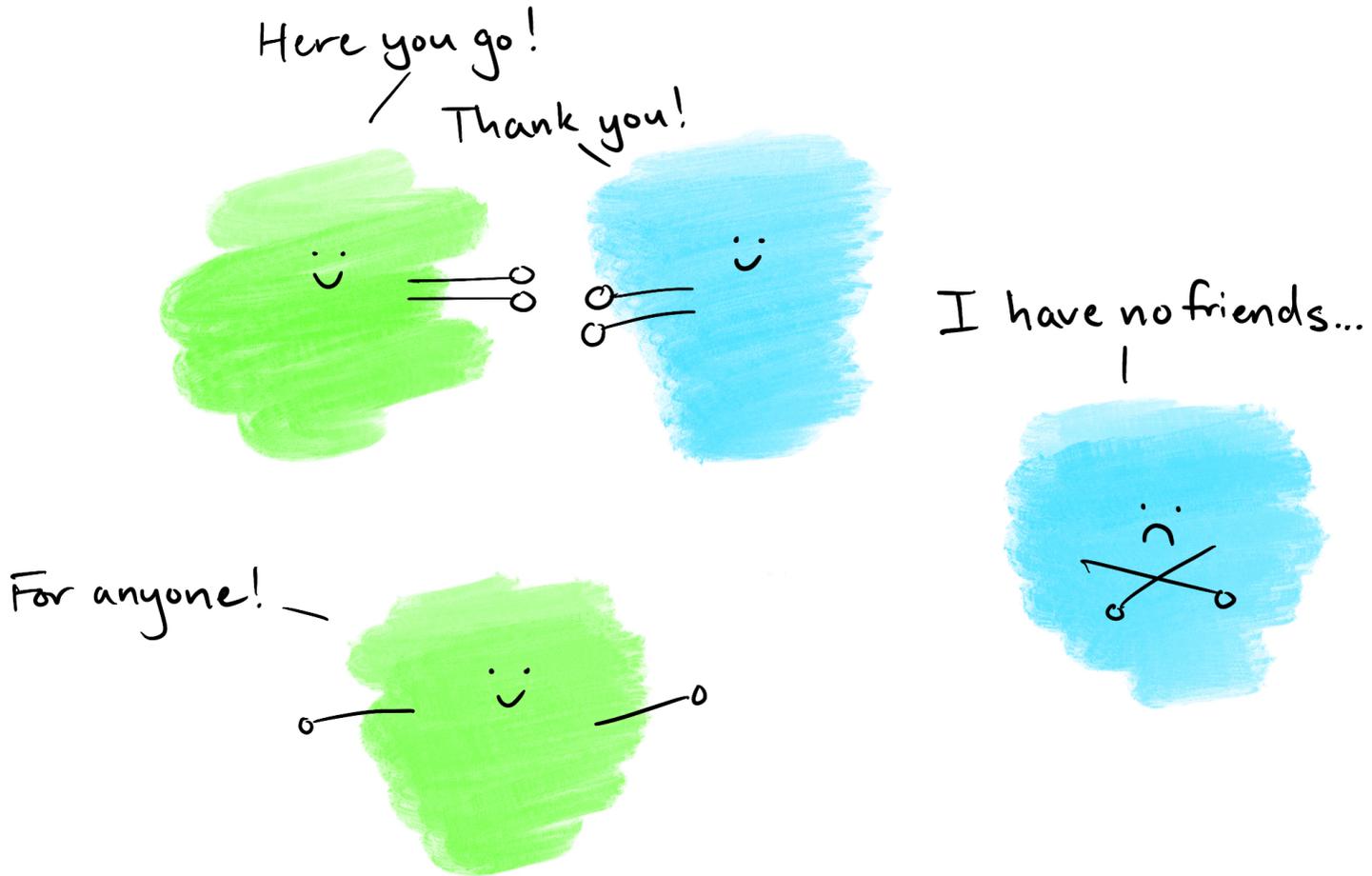
2. Brenda is a blob. Brenda is not a Bob.



3. Therefore, not all blobs are Bobs.



Axiom 6: Creatures are happy **if and only if** they are generous.
(All happy creatures are generous. All generous creatures are happy.)



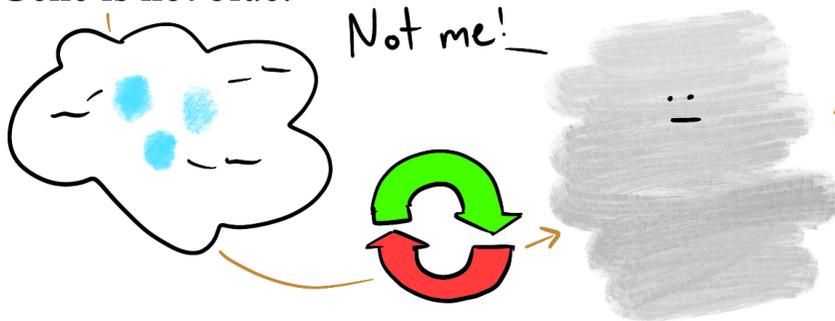
Axiom 7: All unhappy creatures are blue.

Proposition 10: All Genes are happy.

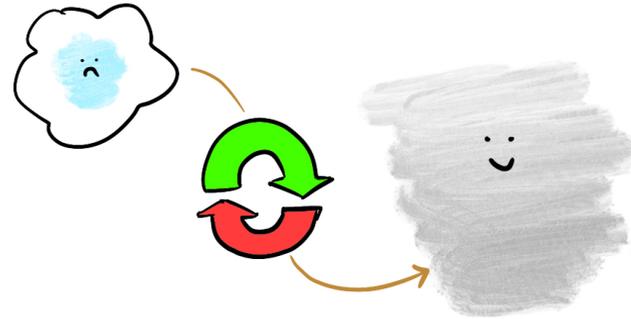
1. Suppose this is Gene.



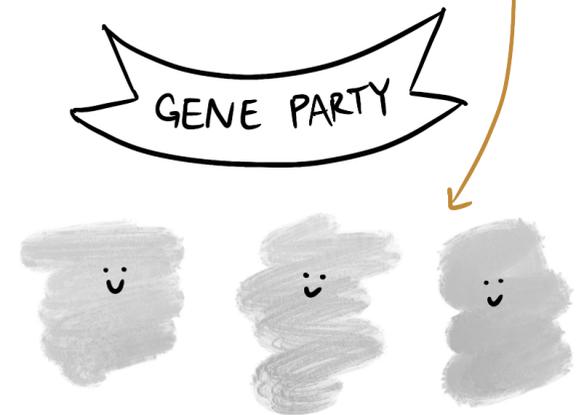
2. All blue creatures have names starting with B. By **contraposition**, all creatures with non-B names are not blue. Therefore, Gene is not blue.



3. All unhappy creatures are blue. By **contraposition**, all non-blue creatures are happy. Therefore, Gene is happy.

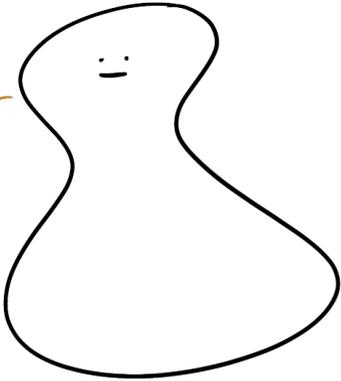


4. Therefore, all Genes are happy.

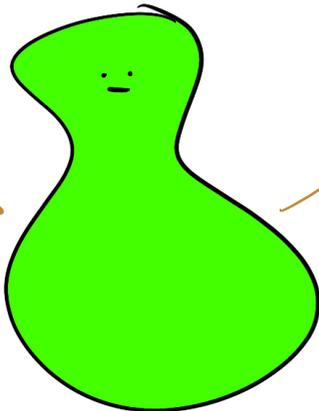
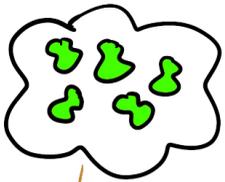


Proposition 11: All glogs are generous.

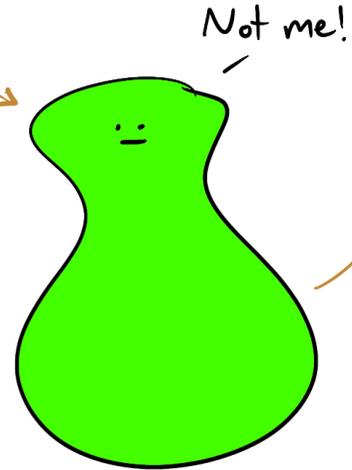
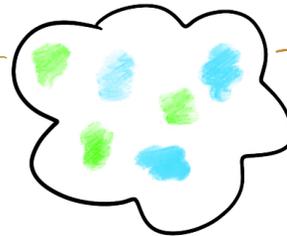
1. Suppose X is a glog.



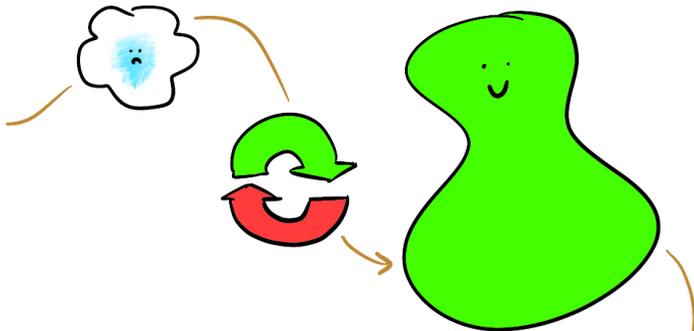
2. Since all glogs are green, X is green.



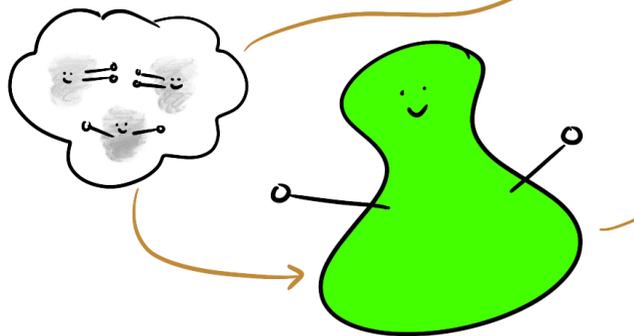
3. Since no creature can be both blue and green, X is not blue.



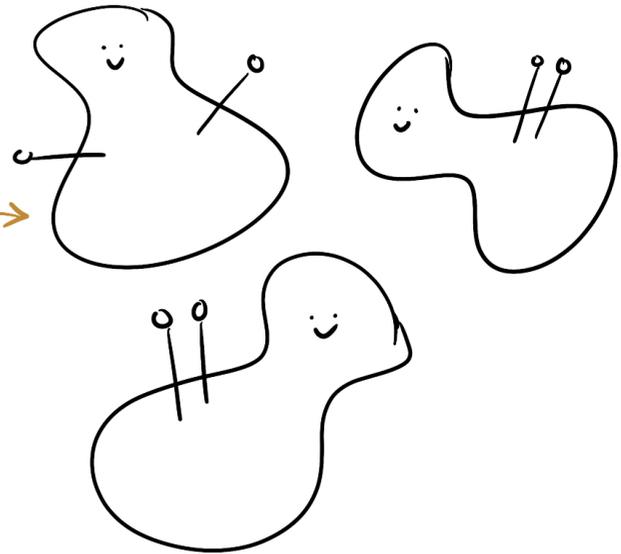
4. All unhappy creatures are blue. By **contraposition**, all non-blue creatures are happy. Therefore, X is happy.



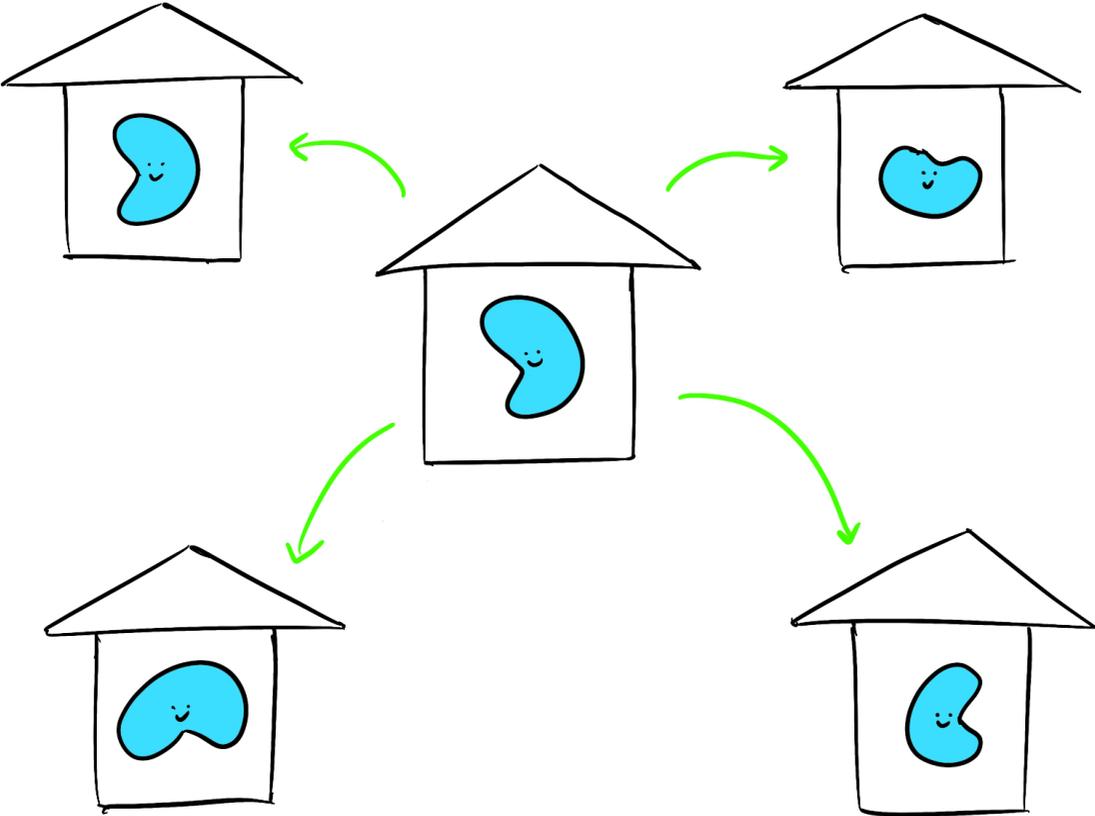
5. Since all happy creatures are generous, X is generous.



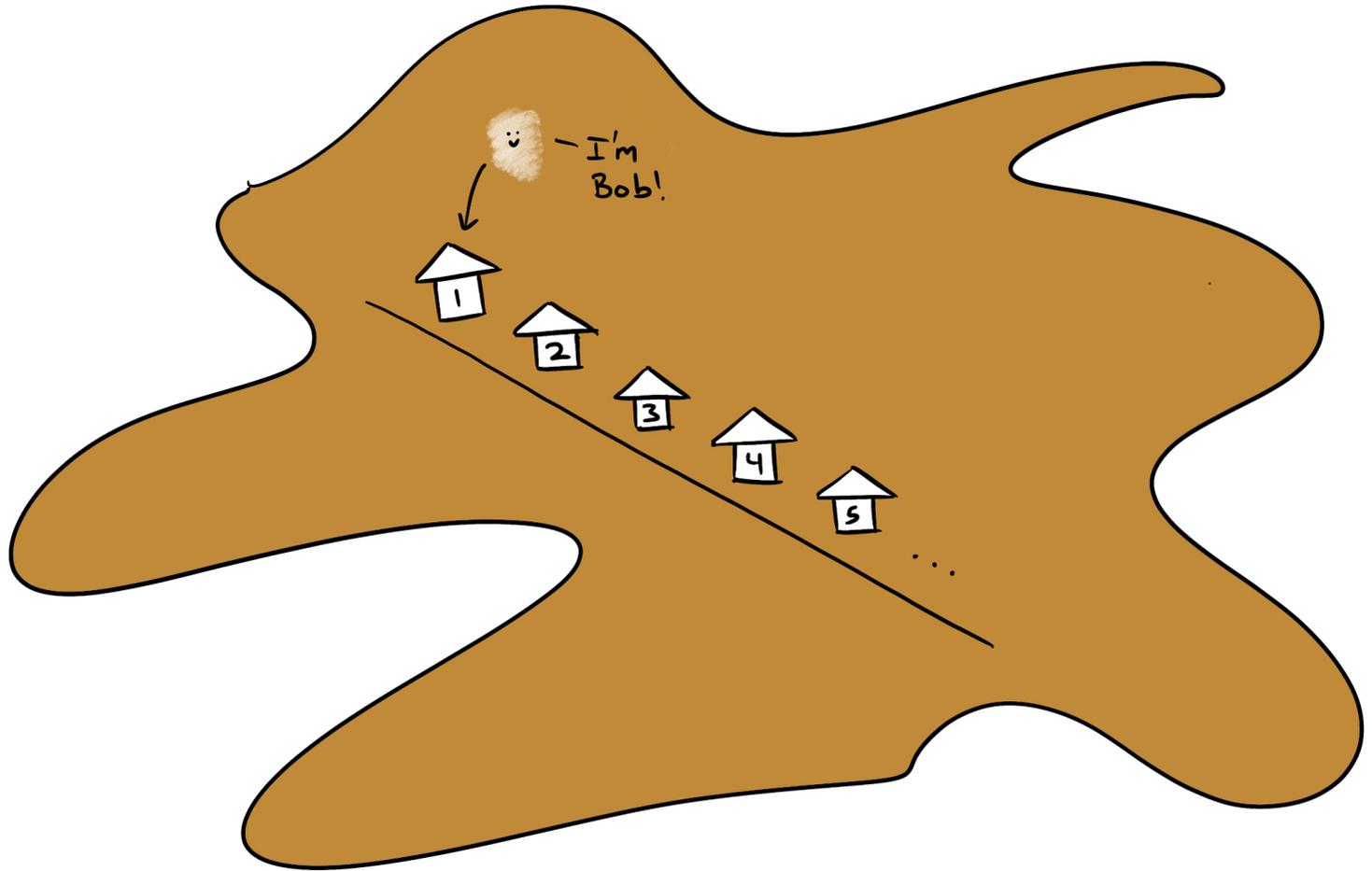
6. Therefore, all glogs are generous.



Axiom 8: All neighbors of blobs are blobs.



Axiom 9: Bog is a neighborhood made of one contiguous street.



Axiom 10: Bob lives in House 1.

Proposition 12: Every resident of Bog is a blob.

Tool: **Induction**



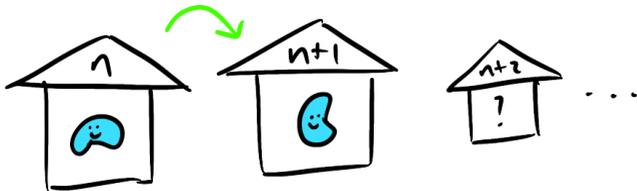
Step 1 (Base Case):
Prove the resident of House 1
is a blob.



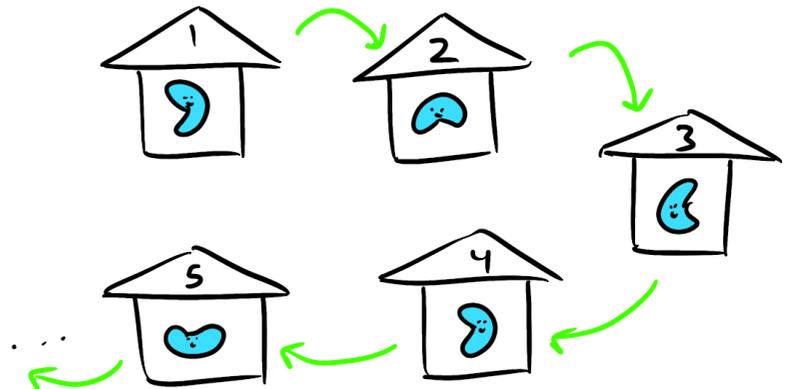
Step 2 (Inductive Hypothesis):
Assume that for some n , the
resident of House n is a blob.



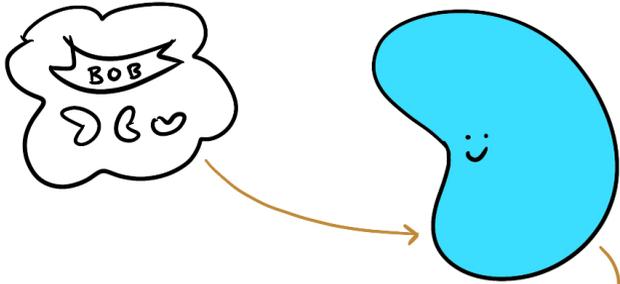
Step 3 (Inductive Step):
Prove the resident of House $n+1$
is also a blob.



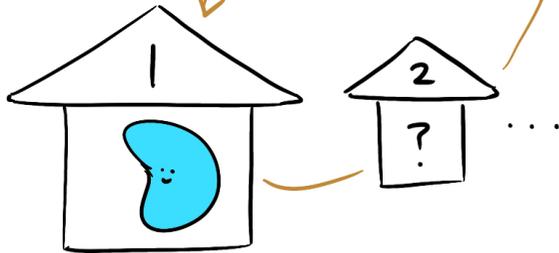
Then every resident is a blob.



1. Since all Bobs are blobs, Bob is a blob.



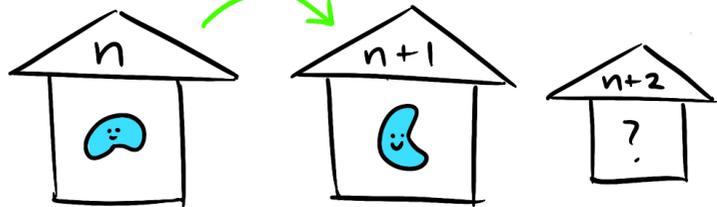
2. Then, the resident of House 1 is a blob. If there's only one resident, we're done. So suppose there's more than 1 resident.



3. Assume that for some number $n <$ the population of Bog, the resident of House n is a blob.



4. Since all neighbors of blobs are blobs, the resident of House $(n+1)$ is also a blob.



5. The 1st resident of Bog is a blob.

If the n th resident is a blob, then the $(n+1)$ th resident is a blob.

Therefore, no matter how many people live in Bog, every resident is a blob.

