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Name:

LINEAR ALGEBRA AND VECTOR ANALYSIS

MATH 22A

Total:

Unit 41: Final Exam Review Lecture

Problems

Problem 41R.1) (10 points):
The graph G in Figure 1 represents a surface in which all triangles are oriented counterclockwise. We are given a 1-form = vector field F on G.
a) (3 points) Find the line integral of F along the boundary curve oriented counter clockwise.
b) (3 points) Enter the curl dF values in the triangles.
c) (2 points) What is the sum of all curl values?
d) (2 points) Why are the results in a) and c) the same?

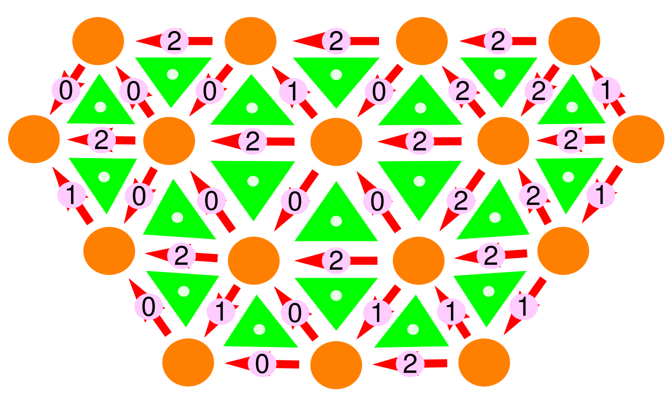

FIGURE 1. A graph analogue to a 2-dimensional region with a 1-form F which models a vector field.

Problem 41R.2) (10 points) Each question is one point:

a) Assume F is a 1-form on a graph $G = (V, E)$. Define $f = d^*F$. Can you say something about $\sum_{x \in V} f(x)$?

b) The volume $V(S_n)$ of the n-dimensional sphere S_n has the property that $V((S_n) \to ...$

c) Who wrote the book "How to solve?" and who invented differential forms?

d) What is $(1+i)^i$?

e) Green was not only doing mathematics, he had an other profession. Which one? Stokes theorem appeared first in an exam problem. Who was one of the pupils?

f) If B and C are row reductions of the same matrix A. What can you say about the length $|B - C|$?

g) We cited a Harvard professor who invoked the anthropic principle to exclude perpetual motion. Who was this? Also and unrelated: who found first the formula for the volume of the sphere?

h) In the relief below in Figure 2, we see the level curves of some function f. Is the function f a Morse function?

i) True or false? There is a non-zero function $f(x)$ which can be differentiated infinitely many times everywhere which has the property that all derivatives at 0 are 0.

j) What is the name of the lantern which approximates a cylinder but for which the surface area explodes?

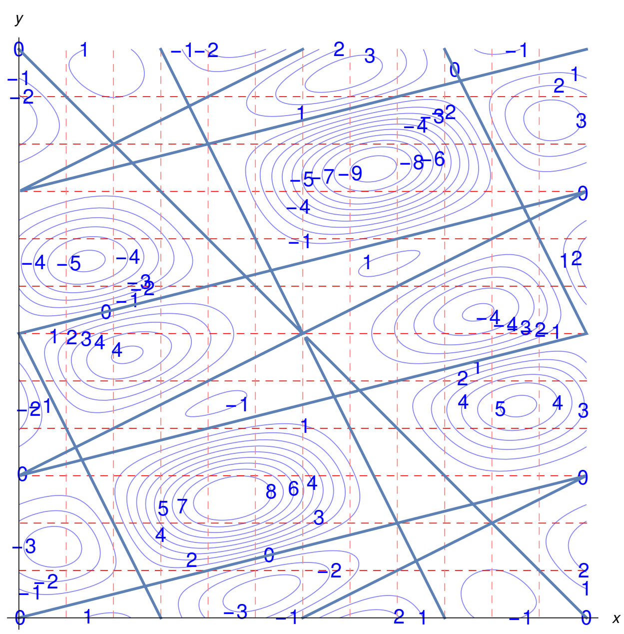

FIGURE 2. Contour map of some function $f(x, y)$.

Problem 41E.3) (10 points) Each problem is 1 point:
a) What is the value of the Hessian determinant $D = \det(d^2 f(x))$ at a critical point x of a Morse function f?
b) What is the curl of a vector field F which is conservative?
c) Take the unit sphere and drill a hole of radius 1/10 from the surface to the center. Is the solid simply connected?
d) One of the three following identities is not defined. F are vector fields in $\mathbb{R}^3$. Which one?
A) $\text{curl}(\text{curl}(F))$,
B) $\text{grad}(\text{div}(F))$,
C) $\text{div}(\text{div}(F))$.
e) Which of the following three vector fields can not be the curl of an other vector field?
A) $F = [x, y, z]$,
B) $F = [y, z, x]$,
C) $F = [z, x, y]$.
f) Which of the following vector fields is a gradient field?
A) $F = [x, y, z]$,
B) $F = [y, z, x]$,
C) $F = [z, x, y]$?
g) What is the exterior derivative dF if $F = xdy + zdz + xdx$?
h) Is it true that $\text{curl}(F)$ is always perpendicular to F?
i) Is there a differentiable function which violates the Fubini theorem? Is there a differentiable function which violates the Clairaut theorem?
j) What is the name of the partial differential equation $\text{curl}(E) = -B_t$?

Problem 41R.4) (10 points):
a) Find the equation $ax + by + cz = d$ of the plane which contains both the line $r(t) = [2 - t, t + 1, 3t]$ as well as the point $P = (3, 5, 1)$.
b) What is the distance from P to the line?

Problem 41R.5) (10 points):
a) Find the critical points of the function $f(x, y) = xy + x^2 + 2x$ and classify them using the second derivative test.
b) Does f have a global mazimum or minimum?

Problem 41R.6) (10 points):
Use the Lagrange method to find the maximum of xyz under the constraint $x + y + z - yz = 1$.

Problem 41R.7) (10 points):
The surface $f(x, y, z, w) = x^2 + y^2 + z^2 - w^2 = 1$ is called a **hyper-hyperboloid**. a) Find the tangent plane at the point $(1, 0, 1, 1)$.
b) The tangent plane has the form $ax + by + cz + dw = e$. Parametrize this plane

Problem 41R.8) (10 points):
Estimate the cube root of $1001 * 999^2$ using a quadratic approximation of $f(x, y) = (xy^2)^{1/3}$ at a suitable point.

Problem 41R.9) (10 points):
a) We live in 22 dimensional space and observe a planet moving along a path $r(t)$ experiencing the acceleration $r''(t) = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$. The planet is initially at the point $r(0) = [10, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$ and has zero initial velocity. Were is it at time $t = 1$?
b) What is the curvature $|T'(0)|/|r'(0)|$ at $t = 0$?

Problem 41R.10) (10 points):
a) Integrate the function $f(x, y) = y$ over the region given in polar coordinates given as $0 \leq r \leq \theta$.
b) Integrate the function $f(x, y, z) = 3 + x + y$ over the solid given in spherical coordinates as $0 \leq \rho \leq \phi$.

Problem 41R.11) (10 points):
a) What is the volume of the solid $G : x^4 + y^4 - 1 < z < 1 + 2x^2 + 2y^4$, $|x|^2 < 1, |y|^2 < 1$?
b) What is the average height $\iiint_G z \, dV / \iint_G 1 \, dV$?

Problem 41R.12) (10 points):
Compute the line integral of the field $F = [y^2 + x, y^5, z^3]$ along the path $r(t) = [t + \sin(t), 0, \sin(4t)]$ from $t = 0$ to $t = \pi$.

Problem 41R.13) (10 points):
Find the line integral of the vector field $F(x, y) = [x^{10} + y, y + \sin(\sin(y))]$ along the triangle $C = ABC$ with $A = (0, 0), B = (2, 0), C = (0, 1)$ in the order $A \to B \to C \to A$.

Problem 41R.14) (10 points):
What is the flux of the vector field $F[x, y, z, w] = [x^3, y^3, z^3, w^3]$ through the sphere $x^2 + y^2 + z^2 + w^2 = 1$ oriented outwards? Remember that we can parametrize the four dimensional ball E as $r(\rho, \phi, \theta_1, \theta_2) = [\rho\cos(\phi)\cos(\theta), \rho\cos(\phi)\sin(\theta), \rho\sin(\phi)\cos(\theta), \rho\sin(\phi)\sin(\theta)]$ with $0 \leq \rho \leq 1, 0 \leq \phi \leq \pi/2, 0 \leq \theta_1 \leq 2\pi, 0 \leq \theta_2 \leq 2\pi$.

Problem 41R.15) (10 points):
What is the flux of the curl of the vector field $F[x, y, z, w] = [xyx, x + y^4wx, -y+x, x*w]$ through the disk surface $r(u, v) = [0, u, v, 0]$, $u^2+v^2 \leq 1$.