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Name:

LINEAR ALGEBRA AND VECTOR ANALYSIS

MATH 22A

Total:

Unit 13: Hourly 1 (II actual hourly)

Problems

Problem 13.1 (10 points):
Prove that
$$1 + 2 + 4 + 8 + \cdots + 2^n = 2^{n+1} - 1$$
for every positive integer n.

Problem 13.2 (10 points):

a) (5 points) Row reduce the matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \end{bmatrix}$.

b) (5 points) Compute the matrix product $\begin{bmatrix} 3 & 4 & 5 \end{bmatrix} A \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

Problem 13.3 (10 points):
a) (2 points) Parametrize the curve $x = \sin(y)$ in $\mathbb{R}^2$.
b) (2 points) Parametrize the curve $r = \sin^2(5\theta)$ in $\mathbb{R}^2$.
c) (2 points) Parametrize the curve $y = x^5 + x, z = 4$ in $\mathbb{R}^3$.
d) (2 points) Parametrize the line $2x + y = 4$ in $\mathbb{R}^2$.
e) (2 points) Parametrize the ellipse $(x-1)^2 + \frac{y^2}{4} = 1$ in $\mathbb{R}^2$.

Problem 13.4 (10 points):
Find the arc length of the curve

$$r(t) = \begin{bmatrix} e^t \\ e^{-t} \\ \sqrt{2}t \end{bmatrix}$$

for $0 \leq t \leq 1$.

Problem 13.5 (10 points):
a) (2 points) Formulate the Cauchy-Schwarz inequality.
b) (2 points) What formula gives the area of the parallelogram spanned by two vectors v and w?
c) (2 points) What formula gives the volume of a parallelepiped spanned by three vectors u, v, w?
d) (2 points) Who invented the quaternions?
e) (2 points) Assume $\text{rref}(A) = \text{rref}(B)$. Does this mean $A = B$?

Problem 13.6 (10 points):
a) (2 points) Write the complex number $z = e^{-i\pi/2}$ in the form $z = a + ib$.
b) (2 points) Which point (x, y, z) has the cylindrical coordinates $(r, \theta, z) = (1, \pi/2, 0)$?
c) (2 points) What are the spherical coordinates (ρ, ϕ, θ) of the point $(x, y, z) = (\sqrt{2}, \sqrt{2}, -2)$?
d) (2 points) What surface is $\rho \sin^2(\phi) = \cos(\phi)$? Give the name and write it in Cartesian coordinates
e) (2 points) What surface is given in cylindrical coordinates by the equation $r \sin(\theta) = 2$?

Problem 13.7 (10 points):

a) (5 points) You are given $r''(t) = \begin{bmatrix} 0 \\ 3 \\ t \end{bmatrix}$ and $r(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and $r'(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. Find $r(1)$.

b) (5 points) What is the curvature of $r(t) = \begin{bmatrix} \cos(t) \\ \sin(t) \\ t \end{bmatrix}$ at $t = 0$?

Problem 13.8 (10 points):

a) (2 points) Find a parametrization of the cone $x^2 + y^2 = z^2$.
b) (2 points) Find a parametrization of $x^2/4 + y^2/9 + z^2/16 = 1$.
c) (2 points) Find a parametrization of the surface $x^2 - y^2 = z$.
d) (2 points) Find a parametrization of the plane $z = 2$.
e) (2 points) Find a parametrization of the cylinder $x^2 + z^2 = 1$.

Problem 13.9 (10 points):
a) (5 points) Find the dot product $A \cdot B = \text{tr}(A^T B)$ between the two matrices

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}.$$

b) (5 points) Find the cosine of the angle between these two matrices.

Problem 13.10 (10 points):
a) (5 points) What is the Jacobian matrix df of the coordinate change

$$f(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} 2x - y + \sin(x) \\ x \end{bmatrix}.$$

b) (5 points) What is the distortion factor $\det(df)$ of the map f which by the way is called the **Chirikov map**.