

Homework 28: Differential equations

This homework is due on Monday, April 16, respectively on Tuesday, April 17, 2018.

- 1 a) Find the general solution to $f' = 30 \log(t) + 20e^{-5t}$.
b) Find the general solution to $f'' = 1/\sqrt{1+t^2}$.
- 2 a) Find the solution to $f' - 5f = e^{7t}$ by inverting the operator $D - 5$.
b) Find the solution again, this time with the cookbook: find first the solution of $f' - 5f = 0$, then make a guess for a special solution.
- 3 a) Find the general solution of $f'' + 225f = e^{44t} + t$.
b) Find the general solution of $f'' + 225f = \cos(15t)$.
c) Find the general solution of $f'' + 225f = \cos(10t)$.
d) Why does the solution in b) behave differently than the solution in c).
- 4 a) Find the general solution to $f'' - 10f' + 25f = 500t$.
b) Find the general solution to $f'' - 10f' + 25f = 4e^{5t}$
- 5 a) Find the general solution to $f''' - 2f'' - f' + 2f = \sin(3t)$.
b) Find the general solution to $f''' - 2f'' - f' + 2f = e^t + e^{-t}$.

Differential equations

To solve a differential equation $(D - 3)f = 0$ we write it as $f' = 3f$ and know that $f = Ce^{3t}$ is a solution, where C is a constant. In this section, we use the variable t and write $f(t)$ as $f(t)$ often represents a quantity which changes in time. We write $f'(t) = d/dt f(t)$ or Df to simplify writing. To solve a differential equation like $(D^2 + 2D - 15)f = 0$, we factor the polynomial and write $(D - 3)(D + 5)f = 0$ which has solutions satisfying $(D + 5)f = 0$ and $(D - 3)f = 0$. The linear space of solutions to $(D - 3)(D + 5)f = 0$ is therefore given by functions $f(t) = Ae^{-5t} + Be^{3t}$, where A, B are constants. To find the solution to the inhomogeneous equation like $(D - 3)f = e^t$, we can invert the operator $D - \lambda = D - 3$. Its inverse is explicitly given by

$$f(t) = Ce^{\lambda t} + e^{\lambda t} \int_0^t e^{-\lambda s} g(s) ds .$$

In the case $g(t) = e^t$, this gives $f(t) = Ce^{3t} + e^t/2$. To solve $(D - 3)(D + 5)f = e^t$, we have now to solve $(D + 5)f = Ce^{3t} + e^t/2$ and therefore invert $D + 5$ with the above formula. We get the general solution $f(t) = C_1 e^{3t} + C_2 e^{-5t} + (-1/12)e^t$. This general operator method works universally, but it can be a bit tedious. In a handout, we show how to get a special solution using a “cookbook recipe”. This is how engineers solve the systems.