# Homework 23: Symmetric matrices

This homework is due on Monday, April 2, respectively on Tuesday, April 3, 2018.

**1** Give a reason why its true or provide a counterexample.

a) The product of two symmetric matrices is symmetric.

b) The sum of two symmetric matrices is symmetric.

c) The sum of two anti-symmetric matrices is anti-symmetric.

d) The inverse of an invertible symmetric matrix is symmetric.

e) If $B$ is an arbitrary $n \times m$ matrix, then $A = B^T B$ is symmetric.

f) If $A$ is similar to $B$ and $A$ is symmetric, then $B$ is symmetric.

g) $A = SBS^{-1}$ with $S^T S = I_n$, $A$ symmetric $\Rightarrow$ $B$ is symmetric.

h) Every symmetric matrix is diagonalizable.

i) Only the zero matrix is both anti-symmetric and symmetric.

**2** Find all the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 2019 & 2 & 3 & 4 & 5 \\ 2 & 2022 & 6 & 8 & 10 \\ 3 & 6 & 2027 & 12 & 15 \\ 4 & 8 & 12 & 2034 & 20 \\ 5 & 10 & 15 & 20 & 2043 \end{bmatrix}.$$

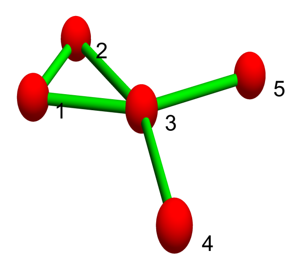
**3** a) Find the eigenvalues and orthonormal eigenbasis of $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$.

b) Find $det(\begin{bmatrix} 7 & 2 & 2 & 2 & 2 \\ 2 & 7 & 2 & 2 & 2 \\ 2 & 2 & 7 & 2 & 2 \\ 2 & 2 & 2 & 7 & 2 \\ 2 & 2 & 2 & 2 & 7 \end{bmatrix})$ using eigenvalues

4 Group the matrices which are similar.

$$A=\begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} B=\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} C=\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} D=\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

5 Find the eigenvalues and eigenvectors of the Laplacian of the Bunny graph. The Laplacian of a graph with $n$ nodes is the $n \times n$ matrix $A$ which for $i \neq j$ has $A_{ij} = -1$ if $i, j$ are connected and 0 if not. The diagonal entries $A_{ii}$ are chosen so that each row add up to 0. $A = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ -1 & -1 & 4 & -1 & -1 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix}$




## Symmetric matrices

$A$ is **symmetric** if $A^T = A$ and **anti-symmetric** if $A^T = -A$. Projections or reflections are symmetric. Symmetric matrices appear in physics or statistics: observables like energy, position or momentum matrices are symmetric, correlation matrices are symmetric. In multi-variable calculus the Hessian matrix consisting of the second derivatives is symmetric. The spectral theorem tells that a symmetric matrix has real eigenvalues, that it has an orthonormal eigenbasis and that can be diagonalized as $B = S^{-1}AS$ with an orthogonal matrix $S$.