

Homework 16: Determinants

This homework is due on Friday, March 9, respectively on Tuesday, March 20, 2018.

- 1 Find the determinants of A, B, C :

$$A = \begin{bmatrix} 0 & 5 & 7 & 3 & 7 & 1 \\ 6 & 0 & 0 & 0 & 0 & 1 \\ 0 & 4 & 0 & 3 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 6 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 3 \\ 3 & 3 & 0 & 0 & 6 & 0 \\ 4 & 2 & 0 & 4 & 0 & 0 \\ 5 & 3 & 2 & 0 & 0 & 0 \\ 6 & 3 & 0 & 0 & 4 & 0 \\ 7 & 0 & 5 & 0 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 2 & 2 & 5 \\ 0 & 0 & 0 & 3 & 1 & 1 & 2 \\ 0 & 0 & 4 & 7 & 3 & 4 & 7 \\ 0 & 5 & 9 & 6 & 4 & 8 & 2 \\ 6 & 8 & 6 & 8 & 0 & 9 & 1 \end{bmatrix}$$

- 2 Without doing much computation, determine whether the following determinant is positive, zero or negative:

$$\begin{bmatrix} 21 & 20^9 & 7 & -6 & 3 & 9 \\ 20^9 & 3 & 2 & 2 & 2 & 2 \\ 6 & 4 & 91 & 1 & 20^9 & -1 \\ 2 & 2 & 20^9 & 1 & -5 & 9 \\ 9 & 1 & -1 & 20^9 & 2 & 2 \\ 7 & 4 & -1 & 2 & 4 & 20^9 \end{bmatrix}.$$

- 3 a) Use the Leibniz definition of determinants to show that the **partitioned matrix** satisfies $\det \begin{bmatrix} A & C \\ 0 & B \end{bmatrix} = \det(A)\det(B)$.
- b) Assume now that A, B are $n \times n$ matrices. Can you find a formula for $\det \begin{bmatrix} 0 & A \\ B & 0 \end{bmatrix}$? (It will depend on n .)
- c) Show that number of up-crossings of a pattern is the same if the pattern is transposed and that therefore $\det(A^T) = \det(A)$.

4 Find the determinant of the matrix $A_{ij} = 2^{ij}$ for $i, j \leq 4$. It is

$$\begin{bmatrix} 2 & 4 & 8 & 16 \\ 4 & 16 & 64 & 256 \\ 8 & 64 & 512 & 4096 \\ 16 & 256 & 4096 & 65536 \end{bmatrix}.$$

First scale some rows to make the computation more manageable.

5 Use Laplace expansion to find a formula for the determinant of the 5×5 matrix $L(5)$ which has 2 in the diagonal and 1 in the side diagonals and 0 everywhere else. Find first the determinants

of $L(2), L(3), L(4)$ and then $L(5) = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$. Can you

see how it continues? Optional: If you dare, try to prove your conjecture.

Determinants I

The **determinant** of a $n \times n$ matrix A is defined as the sum $\sum_{\pi} (-1)^{|\pi|} A_{1\pi(1)} A_{2\pi(2)} \cdots A_{n\pi(n)}$, where π is a permutation of $\{1, 2, \dots, n\}$ and $|\pi|$ is the number of up-crossings in the pattern given by π . This is the Leibniz definition of determinants. By grouping the patterns according of the position in the first column, we get immediately the Laplace expansion $\det(A) = (-1)^{1+1} A_{11} \det(B_{i1}) + \cdots + (-1)^{1+n} A_{n1} \det(B_{n1})$, where for each entry a_{j1} in the first column form the $(n-1) \times (n-1)$ matrix B_{j1} in which the first column and the j 'th row of A are deleted.