Homework 15: Data fitting

This homework is due on Wednesday, March 7, respectively on Thursday, March 8, 2018.

1 a) Find the least square solution x^* of the system $Ax = b$ with $A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \\ 2 & -4 \end{bmatrix}$, and $b = \begin{bmatrix} 125 \\ 125 \\ 125 \end{bmatrix}$.

b) What is the matrix P which projects on the image of A?

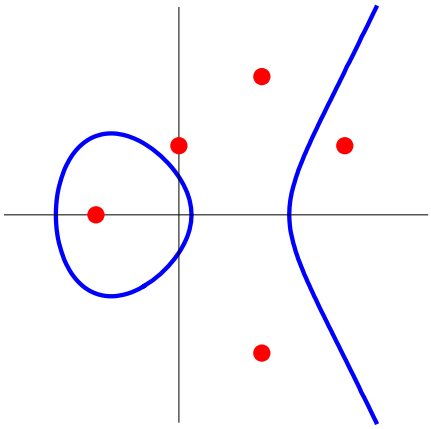
2 Find the function $y = f(x) = 1 + ax + bx^3$, which best fits the data

x	y
-1	5
2	4
-2	8

3 A curve of the form

$$y^2 = x^3 + ax + b$$

is called an **elliptic curve** in Weierstrass form. Elliptic curves are important in cryptography. Use data fitting to find the best parameters (a, b) for an elliptic curve given the following points:



$$\begin{aligned} (x_1, y_1) &= (1, 2) \\ (x_2, y_2) &= (-1, 0) \\ (x_3, y_3) &= (2, 1) \\ (x_4, y_4) &= (0, 1) \end{aligned}$$

We leave the comfort zone and look at extreme cases.

a) Analyze the best linear fit $f(x) = ax + b$ for the three data points $(1,1), (1,2), (1,3)$.

4 b) To find the best linear $f(x) = a + bx$ for the four data points $(1,1), (1,1), (2,2), (4,7)$ with $f(x) = a + bx$, we shorten the climb by taking the points $(1,1), (2,2), (4,7)$ as one point was redundant. Do we get the same?

David Lama climbs in 2015 Avaatara 9a in the Baatara Gorge Lebanon.

5 Find the circle $a(x^2+y^2)+b(x+y) = 1$ which best fits the data

x	y
0	1
-1	0
1	-1
1	1

In other words, find the least square solution for the system of equations for the unknowns a, b which aims to have all 4 data points (x_i, y_i) on the circle.

Data fitting

Given a system $Ax = b$. Any solution of $(A^T A)x = A^T b$ is called a **least square solution** (these always exist). (Reason: solve $A^T(Ax - b) = 0$ for x, assuring that $Ax - b$ is perpendicular to $\mathrm{im}(A)$.) The least square solution is unique if A has a trivial kernel. In that case $x = (A^T A)^{-1} A^T b$. The matrix $A(A^T A)^{-1} A^T$ is now the projection matrix onto $\mathrm{im}(A)$. If the columns of A are orthonormal, this simplifies to $P = AA^T$.