# Homework 10: Coordinates

This homework is due on Wednesday, February 21, respectively on Tuesday, February 20, 2018.

**1** What are the $\mathcal{B}$-coordinates of the vector $\vec{v}$ in the basis $\mathcal{B}$.

$$\vec{v} = \begin{bmatrix} 7 \\ 3 \\ 4 \\ 5 \end{bmatrix}, \mathcal{B} = \{ \begin{bmatrix} 3 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \} \text{ ?}$$

**2** What is the matrix $B$ for the transformation $A = \begin{bmatrix} 3 & 1 \\ -1 & 4 \end{bmatrix}$ in the basis $\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \}$.

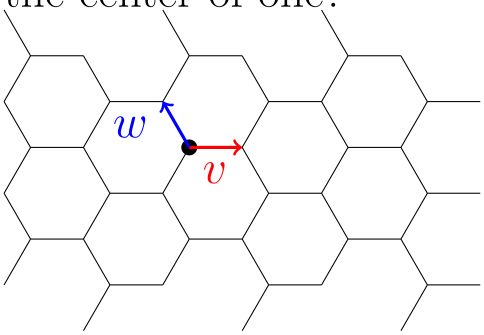
**3** Chose a suitable basis to solve the following two problems:
a) Find the matrix $A$ which belongs to a reflection at the plane $3x + 3y + 6z = 0$.
b) Find the matrix $A$ which belongs to the reflection at the line spanned by $\begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$.

**4** Find the matrix $A$ corresponding to the orthogonal projection onto the plane spanned by the vectors $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$.

**5** The whole plane is covered with regular hexagons "Graphene", where the first basis vector is $v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. a) Find $w$ so that $\mathcal{B} = \{v, w\}$ is the basis as seen in the picture.

b) What are the standard coordinates of the vector given in the $\mathcal{B}$ basis as $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$?

c) Is the point with $\mathcal{B}$ coordinate $\begin{bmatrix} 23 \\ 72 \end{bmatrix}$ a vertex of a hexagon or the center of one?




Picture near Harvard School of Design

## Coordinates

Given a basis $\mathcal{B} = \{\vec{v}_1, \dots, \vec{v}_n\}$ of a linear space $V$, every $\vec{w}$ in $V$ can be written as $\vec{w} = c_1\vec{v}_1 + \cdots + c_n\vec{v}_n$, where $c_i$ are the **coordinates** of $v$. The basis defines a matrix $S = \begin{bmatrix} | \\ \vec{v}_1 \\ | \end{bmatrix}, \begin{bmatrix} | \\ \vec{v}_2 \\ | \end{bmatrix} \cdots \begin{bmatrix} | \\ \vec{v}_n \\ | \end{bmatrix}$. Since $S\vec{c} = \vec{w}$ we get $\boxed{\vec{c} = S^{-1}\vec{w}}$. If $A$ is a matrix given in the standard basis $e_1, \dots, e_n$ and $B$ is the matrix written in the basis $\mathcal{B}$, then $\boxed{B = S^{-1}AS}$. We say $B$ is **similar** to $A$. Why do we want to change basis? Because it is convenient: for example if $v_1, v_2$ are non-parallel vectors in a plane and $v_3$ is perpendicular to the plane then a projection onto the plane is the matrix $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. The matrix in the standard basis is then $A = SBS^{-1}$.