

Homework 9: Dimension

This homework is due on Friday, February 16, respectively on Tuesday, February 20, 2018.

- 1 Determine the rank and nullity of the following matrices and verify

that the rank-nullity theorem holds: $A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $B =$

$$\begin{bmatrix} 5 & 4 & 3 & 2 \\ 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 5 & 4 & 3 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}.$$

- 2 a) In each of the 6 cases $k = 2, 3, 4, 5, 6$; give a 4×6 matrix A for which the dimension of the kernel of A is k .
 b) Is there a 4×6 matrix which has a kernel of dimension 1? Explain why or why not.
- 3 Find out whether the following set of vectors related to prime numbers forms a basis of R^4 .

$$\left\{ \begin{bmatrix} 2 \\ 3 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 7 \\ 11 \end{bmatrix}, \begin{bmatrix} 5 \\ 7 \\ 11 \\ 13 \end{bmatrix}, \begin{bmatrix} 7 \\ 11 \\ 13 \\ 17 \end{bmatrix} \right\}$$

4 The following matrices encode the letters of "Touchy":

$$T = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}, O = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}, U = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}, H = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, Y = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

a) Group the letters which have the same kernel.

b) Group the letters which have the same image.

5 Consider the subspace V consisting of all vectors in R^4 which are

perpendicular to both $v = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$ and $w = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$.

Find a basis for V .

Basis

If V is a linear space and $\{v_1, \dots, v_n\}$ is a basis, then n is the **dimension** of V . If $\vec{v}_1, \dots, \vec{v}_p$ are linearly independent in V and $\vec{w}_1, \dots, \vec{w}_q$ span V then $p \leq q$. The dimension of the image of A is called the **rank** of A . The dimension of the kernel of A is called the **nullity** of A . The rank-nullity theorem tells that the sum of the rank and the nullity is equal to the number of columns of A . It's easy to see why this is true if you remember that the rank of A is the number of leading 1's in $\text{rref}(A)$ and the nullity of A is the number of free variables (columns without leading 1).