Homework 6: Matrix Algebra

This homework is due on Friday, February 9, respectively on Tuesday February 13, 2018.

1 For each pair of matrices A and B, compute both AB and BA

a) $A = \begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 2 \\ -1 & 3 \end{bmatrix}$.

b) $A = \begin{bmatrix} 2 & 3 & 4 \\ 2 & -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 2 \\ 2 & 3 \\ 6 & 11 \end{bmatrix}$.

2 a) Find a 2×2 matrix A with no 0 or 2 entries such that $A^2 = 0$.

b) Can you find a 3×3 matrix A with entries $-2, 1$ such that $A^2 = 0$? (You can search computer assisted).

c) Can you find a 4 times 4 matrix with entries $-1, 3$ such that $A^2 = 0$?

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(* See what this does and modify *)
Do[A=Table[RandomChoice[{ -1 ,1}],{4},{4}];
   If[Max[Abs[A.A]]==0,Print[A]],{10000}]
```

3 a) Find the inverse of the matrix A made from the first 4 rows of

Pascal's triangle. $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix}$.

b) The following $0-1$ matrix B has the property that the inverse is

again an integer matrix: $B = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$. Find the inverse.

4 a) Assume $A^8 = A \cdot A \cdot A \cdot A \cdot A \cdot A \cdot A \cdot A$ is the identity matrix. Can you find a simple formula in terms of A which gives A^{-1}?
b) Find a transformation in the plane with A^4 not being the identity such that $A^8 = 1$. What is A^{-1}?

5 a) Assume A is small enough so that $B = 1 + A + A^2 + A^3 + ...$ converges. Verify that B is the inverse of $1 - A$. (Leontief).
b) Use Mathematica to plot A^{-1} for the 100 x 100 matrices defined by $A_{nm} = n^2 + 1.1m$.
c) Use Mathematica to plot A^{-1} for the 100 x 100 matrix $A_{n,m} = gcd(n,m)$, the greatest common divisor. (If you can explain this pattern, this might be a research paper. It seems unexplored).

```
MatrixPlot[Table[n^2+1.1 m,{n,300},{m,300}]];
```

Matrix Algebra

Matrices can be added, multiplied with a scalar. One can also form the product of two matrices $A \cdot B$ as well as the inverse matrix A^{-1} if the matrix is invertible. These operations constitute the **matrix algebra**. It behaves like the algebra of real numbers but the multiplication is no more commutative in general. Besides the matrix 0 where all entries are zero there are other matrices which are not invertible. We write 1 for the identity matrix which has 1 in the diagonal and 0 everywhere else. Now $A1 = A$.

```
A={{5,2},{3,4}};
Inverse[A]+MatrixPower[A,7] + IdentityMatrix[2]
```