

Homework 17: Determinants II

This homework is due on Monday, March 20, respectively on Tuesday, March 21, 2017. Its a good idea to finish this before spring break!

- 1 a) We find here the determinant of the 5×5 matrix A for which the entry A_{km} is $\phi(k + m)$, where ϕ is the Eulertotient function giving the number of positive integers less than n that are coprime to n . Use row reduction to find the determinant:

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 2 \\ 2 & 2 & 4 & 2 & 6 \\ 2 & 4 & 2 & 6 & 4 \\ 4 & 2 & 6 & 4 & 6 \\ 2 & 6 & 4 & 6 & 4 \end{bmatrix}.$$

- b) Find the determinant of the Euler totient matrix of size 1000×1000 . The result has only 2505 digits. What is the last digit? Excessive homework? No! You have all spring break ...

Hint: use a machine:

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M=1000; A=Table[EulerPhi[n+k],{n,M},{k,M}];
result=Det[A]; Last[IntegerDigits[result]]
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- 2 a) Find the determinant of

$$B = \begin{bmatrix} 4 & 3 & 0 & 0 & 0 & 0 \\ 4 & 4 & 3 & 0 & 0 & 0 \\ 4 & 4 & 4 & 3 & 0 & 0 \\ 4 & 4 & 4 & 4 & 3 & 0 \\ 4 & 4 & 4 & 4 & 4 & 3 \\ 4 & 4 & 4 & 4 & 4 & 4 \end{bmatrix}.$$

- b) Find the determinant of $7B$.

- 3 a) Find the determinant of

$$A = \begin{bmatrix} 3 & 1 & 1 & 2 & 2 & 2 \\ 0 & 0 & 1 & 2 & 2 & 2 \\ 0 & 6 & 3 & 2 & 2 & 2 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 & 1 & 4 \end{bmatrix}$$

- b) Find the determinant of A^5 .

4 Argue geometrically why the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

has maximal absolute determinant $|\det(A)|$ among all matrices with entries in $\{-1, 1\}$.

5 a) Find A, B such that $\det(A + B) \neq \det(A) + \det(B)$.

b) What values can an orthogonal matrix have?

c) Verify that $|\det(A)|$ only depends on R if $A = QR$ is the QR factorization.

Determinants II

Determinants can be computed using row reduction: If during row reduction m swapping operations have occurred and the scaling factors are c_1, \dots, c_k , then

$$\det(A) = \frac{(-1)^m}{c_1 \cdots c_k} \det(\text{rref}(A))$$

Here are some more properties:

- $|\det(A)|$ is the volume of a parallel epiped
- $\det(AB) = \det(A)\det(B)$
- $\det(A^T) = \det(A)$
- $\det(A^n) = (\det(A))^n$
- $\det(A^{-1}) = 1/\det(A)$