Homework 11: Linear spaces

This homework [1] is due on Friday, February 24, respectively on Thursday, February 23, 2017.

1 Which of the following spaces are linear spaces?

a) All 3×3 matrices for which the diagonal sum is zero.

b) All 3×3 matrices for which the diagonal product is zero.

c) All polynomials of degree exactly 3.

d) All non-invertible 3×3 matrices.

e) All 3×3 matrices for which the diagonal elements are zero.

f) All functions f in $C(R)$ for which $f(5) = 0$.

g) All functions f in $C^\infty(R)$ for which $f''(0) = 1$.

h) All functions f in $C^\infty(R)$ for which $f(0) + f''(2) = 0$.

2 Find a basis for the space of

a) all 2×2 rotation dilation matrices $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$.

b) all 2×2 reflection dilation matrices $\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$.

c) all 2×2 horizontal shears. (Trick question! Why?)

d) all the 2×2 matrices for which Ae_1 is parallel to e_1.

e) all the diagonal 3×3 matrices.

f) all the 2×2 dilation matrices.

3 Find a basis for all the 2×2 matrices A for which

$$A \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

[1] Not much seems to happen in this lecture but we make a giant step. We use the language of linear algebra in realms where it originally was not intended to be used like calculus, where we deal with functions. Or then we can look at a spread sheet or picture or movie or relational database and see it as a vector in a linear space.

4 A function is called **even** if $f(-x) = f(x)$ for all x. A function is called **odd** if $f(-x) = -f(x)$ for all x. Find a basis:
a) for all the even polynomials in $P_4(R)$
b) for all the odd polynomials in $P_4(R)$.
c) for all polynomials f in $P_4(R)$ for which $f(0) = 0$.

5 a) Find a concrete 3×3 matrix A with no zeros for which $A\vec{v} = \vec{0}$, where $\vec{v} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T$.
b) Let V be the set of 3×3 matrices for which $\vec{v}$ is in the kernel. Is this a linear space?

Linear space

A set V in which we can add, scale and which contains a 0 is a **linear space**. To check that V is a linear space, verify (i) 0 is in X, (ii) if x and y are both in X, then $x + y$ is in X, (iii) if x is in X, then λx is in X for all constants c. There are three important classes of linear spaces: first the subspaces of R^n as treated before, the second class is the set $M(n, m)$ of all $n \times m$ matrices. Finally, there is the class $C(R)$ for all continuous functions on the real line. It contains the linear space $C^\infty(R)$ for all smooth functions, functions which can be differentiated arbitrary often. We also write $P_n(R)$ for the set of polynomials of degree less or equal to n. It has dimension $n + 1$. In order to check whether a subset of functions or matrices or vectors in R^n are a linear space, check the three things (i),(ii),(iii). In the case of functions, 0 is the function which is 0 for all x.