

Homework 6: Matrix Algebra

This homework is due on Friday, February 10, respectively on Tuesday February 14, 2017.

1 For each pair of matrices A and B , compute both AB and BA

a) $A = \begin{bmatrix} 7 & 2 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}.$

b) $A = \begin{bmatrix} 2 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 4 & 2 \\ 2 & 3 \\ 6 & 11 \end{bmatrix}.$

2 a) Find a 2×2 matrix A with no 0 entries such that $A^2 = 0$.

b) Use Mathematica to explore whether there is a 3 times 3 matrix with entries $-1, 1$ such that $A^2 = 0$.

c) Find a 4×4 matrix A with entries $-1, 1$ such that $A^2 = 0$. (You can search computer assisted).

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Do[A=Table [ RandomChoice [ { -1 , 1 } ] , { 4 } , { 4 } ] ;  
    If [ Max [ Abs [ A . A ] ] == 0 , Print [ A ] ] ,  
    { 100000 } ]
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3 a) Find the inverse of the matrix A made from the first 5 rows of

Pascal's triangle. $A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 1 & 3 & 3 & 1 & 0 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}.$

b) The following $0 - 1$ matrix B has the property that the inverse

is again a $0 - 1$ matrix: $B = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}.$ Find the inverse.

- 4 a) Assume $A^7 = A \cdot A \cdot A \cdot A \cdot A \cdot A \cdot A$ is the identity matrix. Can you find a simple formula in terms of A which gives A^{-1} ?
- b) Find a transformation in the plane which has the property that $A^7 = 1$. Find A^{-1} .

- 5 a) Assume A is small enough so that $B = 1 + A + A^2 + A^3 + \dots$ converges. Verify that B is the inverse of $1 - A$. (Used by Leontief).

b) Use Mathematica to plot A and B^{-1} for the 300 x 300 matrices defined by $A_{nm} = \cos(nm + 3)$ and $B_{nm} = \sin^3(nm)$.

(* Example to plot inverse of a matrix *)
 $A = \text{Table}[\text{N}[\text{Tan}[n+m]], \{n, 200\}, \{m, 200\}];$
 $\text{MatrixPlot}[\text{Inverse}[A]]$

Matrix Algebra

Matrices can be added, multiplied with a scalar. One can also form the product of two matrices $A \cdot B$ as well as the inverse matrix A^{-1} if the matrix is invertible. These operations constitute the **matrix algebra**. It behaves like the algebra of real numbers but the multiplication is no more commutative in general. Besides the matrix 0 where all entries are zero there are other matrices which are not invertible. We write 1 for the identity matrix which has 1 in the diagonal and 0 everywhere else. Now $A1 = A$.

$A = \{\{5, 2\}, \{3, 4\}\};$ $\text{MatrixInverse}[A]$
 $\text{MatrixPower}[A, 7]$
 $\text{IdentityMatrix}[2] + A$